A Decidable Extension of $\mathcal{SRIQ}$ with Disjunctions in Complex Role Inclusion Axioms

Milenko Mosurović[1], Henson Graves[2], and Nenad Krdžavac[3]

[1] Faculty of Mathematics and Natural Science, University of Montenegro, Podgorica, ul. Džordža Vašingtona bb, 81000 Podgorica, Montenegro.
milenko@ac.me

[2] Algos Associates, 2829 West Cantey Street, Fort Worth, TX 76109 United States
henson.graves@hotmail.com

[3] Department of Accounting and Information Systems, College of Business and Law, University College Cork, Cork City, Ireland.
N.Krdzavac@ucc.ie

Abstract. This paper establishes the decidability of $\mathcal{SR}^{\sqcup}\mathcal{IQ}$ which has composition-based role Inclusion axioms (RIAs) of the form $R_1 \circ \cdots \circ R_n \dot{\sqsubseteq} T_1 \sqcup \cdots \sqcup T_m$. Also the consistency of an Abox $\mathcal{A}$ of $\mathcal{SR}^{\sqcup}\mathcal{IQ}$ DL w.r.t. Rbox $\mathcal{R}$ is established. Motivation for this kind of RIAs comes from applications in the field of manufactured products as well as other conceptual modeling applications such as family relationships. The solution is based on a tableau algorithm.

Keywords: Description Logic, Manufacturing system,Tableau, Composition-based Role Inclusion Axiom.

1 Introduction

Description logic (DL) [1] has focused on extending decidability results to DLs with more complex RIAs [6, 7, 9]. However, the logic $\mathcal{SROIQ}$ DL which is logical basis for the standard Ontology Web Language OWL 2 [3], does not admit assertions which have role unions on the right hand side of RIAs. Many applications involve RIAs with role unions on the right side. For example in modeling an engine in a car that can power wheelInCar or oilPump or generator, or all of these, at the same time [8, 2]. This model can be described in the following composition-based RIAs [11]:

$$engineInCar \circ powers \sqsubseteq wheelInCar \sqcup generatorInCar \sqcup oilPunInCar \quad (1)$$

One can conclude that for an individual car c_1 and an individual p_1: if p_1 is powered by an individual engine e_1 in the car c_1 then p_1 is an individual wheel or a generator or an oilpump in c_1. The RIA of the form (1) ca be expressed in an extension of $\mathcal{ALC}$ DL with composition-based RIAs [11], but $\mathcal{SROIQ}$ DL does not support such composition-based RIAs. Modeling such RIAs in the extensions of $\mathcal{ACL}$ DL considered only two roles on the left hand side of the RIAs. This paper introduces the $\mathcal{SR}^{\sqcup}\mathcal{IQ}$ DL that extends $\mathcal{SRIQ}$ DL [5] with composition-based RIAs of the form (2). As noted in [11] the RIA of the form (2) are not role value-maps [10]. The logic analyzed in this paper overcomes the following shortcomings of the logics studied in [11]:

1. Finite automata handle composition-based RIAs of the form (2).
2. Does not require a Rbox to be admissible [11],
3. Does not require all roles to be disjoint [11],
4. Allows more than two roles on the left hand side of composition-based RIAs.

The rest of the paper is organized as follows. Next section gives definition of $\mathcal{SR}^{\sqcup}\mathcal{IQ}$ DL. Section 3 defines tableau for $\mathcal{SR}^{\sqcup}\mathcal{IQ}$ and proves decidability of the logic. The section also gives and example of tableau for RIA of the form (2). The last section concludes the paper.

2 Preliminaries

The alphabet of $\mathcal{SRIQ}$ and $\mathcal{SR}^{\sqcup}\mathcal{IQ}$ DL consists of set of concept names $\mathcal{N}_C$, set of role names $\mathcal{N}_R$, set of simple role names $\mathcal{N}_S \subset \mathcal{N}_R$ and finally, a set of individual names $\mathcal{N}_I$. The set of roles is $\mathcal{N}_R \cup \{R^-|R \in \mathcal{N}_R\}$ and on this set the function $Inv(\cdot)$ is defined as $Inv(R) = R^-$ and $Inv(R^-) = R$ for $R \in \mathcal{N}_R$. A role chain is a sequence of roles $w = R_1R_2 \dots R_n$.

$\mathcal{SR}^{\sqcup}\mathcal{IQ}$ language is an extension of $\mathcal{SRIQ}$ [5], by allowing new kinds of RIAs in role hierarchy. The syntax of the $\mathcal{SR}^{\sqcup}\mathcal{IQ}$ DL concepts, Rbox, Tbox and Abox are given in definitions 1, 2 and 3 following [5].

Definition 1. *Set of $\mathcal{SR}^{\sqcup}\mathcal{IQ}$ concepts is a smallest set such that*

- *every concept name and $\top$, $\bot$ are concepts, and,*
- *if C and D are concept and R is a role, S is simple role, n is non-negative integer, then $\neg C$, $C \sqcap D$, $C \sqcup D$, $\forall R.C$, $\exists R.C$, $\exists S.Self$, $(\leq nS.C)$, $(\geq nS.C)$ are concepts.*

A general concept inclusion axiom (GCI) is an expression of the form $C \dot{\sqsubseteq} D$ for two $\mathcal{SR}^{\sqcup}\mathcal{IQ}$-concepts C and D. A Tbox $\mathcal{T}$ is a finite set of GCIs.
An individual assertion has one of the following forms: $a : C, (a, b) : R, (a, b) : \neg S$, or $a \dot{\neq} b$, for $a, b \in \mathcal{N}_I$ (the set of individual names), a (possibly inverse) role R, a (possibly inverse) simple role S, and a $\mathcal{SR}^{\sqcup}\mathcal{IQ}$-concept C. A $\mathcal{SR}^{\sqcup}\mathcal{IQ}$-Abox $\mathcal{A}$ is a finite set of individual assertions.

A (composition-based) RIA is a statement of the form [11]:

$$R_1 \cdots R_n \dot{\sqsubseteq} T_1 \sqcup \cdots \sqcup T_m. \tag{2}$$

Without additional restrictions on RIAs, some DLs [11] with composition-based RIAs are undecidable.

Definition 2. *Strict partial order $\prec$ (irreflexive, transitive, and antisymmetric), on the set of roles, provides acyclicity [5]. Allowed RIAs in $\mathcal{SRIQ}$ DL with respect to $\prec$, are expressions of the form $w\dot{\sqsubseteq}R$, where [4, 5]:*

1. *R is a simple role name, $w = S$ is a simple role, and $S \prec R$ or $S = R^-$ or*
2. *$R \in \mathcal{N}_R \backslash \mathcal{N}_S$ is a role name and*
   $w = RR$, or
   $w = R^-$, or
   $w = S_1 \cdots S_n$ and $S_i \prec R$, for $1 \leq i \leq n$, or
   $w = RS_1 \cdots S_n$ and $S_i \prec R$, for $1 \leq i \leq n$, or
   $w = S_1 \cdots S_n R$ and $S_i \prec R$, for $1 \leq i \leq n$

A $\mathcal{SRIQ}$ role hierarchy is a finite set $\mathcal{R}_h^1$ of RIAs. A $\mathcal{SRIQ}$ role hierarchy $\mathcal{R}_h^1$ is regular if there exists strict partial order $\prec$ such that each RIA in $\mathcal{R}_h^1$ is allowed with respect to $\prec$ [4, 5].

Definition 3. *A $\mathcal{SR}^{\sqcup}\mathcal{IQ}$ role hierarchy is a finite set $\mathcal{R}_h = \mathcal{R}_h^1 \cup \mathcal{R}_h^2$, where $\mathcal{R}_h^1$ is $\mathcal{SRIQ}$ role hierarchy and R_h^2 is set of RIA $R_{i1} \cdots R_{in_i} \sqsubseteq T_{i1} \sqcup \cdots \sqcup T_{im_i}$, and T_{ij} are not simple roles, for $i = 1, \ldots, k$. A $\mathcal{SR}^{\sqcup}\mathcal{IQ}$ role hierarchy $\mathcal{R}_h$ is regular if $\mathcal{R}_h^1$ is regular and T_{ij} does not appear on the left hand side of RIAs in $\mathcal{R}_h$. A $\mathcal{SR}^{\sqcup}\mathcal{IQ}$ set of role assertions is a finite set $\mathcal{R}_a$ of the assertions $Ref(R)$, $Irr(S)$, $Sym(R)$, Tra(V), and Dis(T, S), where R is a role, S, T are simple roles and V is not simple role [5]. A $\mathcal{SR}^{\sqcup}\mathcal{IQ}$ Rbox $\mathcal{R} = \mathcal{R}_h \cup \mathcal{R}_a$, where $\mathcal{R}_h$ is $\mathcal{SR}^{\sqcup}\mathcal{IQ}$ role hierarchy and $\mathcal{R}_a$ is a set of role assertions.*

If $\mathcal{R}_h^1$ is regular w.r.t strict partial order $\prec$ then we extend $\prec$ such that $R_{ij} \prec T_{il}$ hold, $i = 1, \ldots, k$ and $j = 1, \ldots, n_i$, $l = 1, \ldots, m_i$. Further, we assume that labels, such as k, n_i, m_i, T_{il}, R_{ij}, have the same meaning as defined in definition 3.

Definition 4. *The semantics of the $\mathcal{SR}^{\sqcup}\mathcal{IQ}$ DL is defined by using interpretation. An interpretation is a pair $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where $\Delta^{\mathcal{I}}$ is a non-empty set, called the domain of the interpretation. A valuation $\cdot^{\mathcal{I}}$ associates: every concept name C with a subset $C^I \subseteq \Delta^{\mathcal{I}}$; every role name R with a binary relation $R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ and, every individual name a with an element $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$ [1].*

Definition 5. *An interpretation $\mathcal{I}$ extends to $\mathcal{SR}^{\sqcup}\mathcal{IQ}$ complex concepts and roles according to the following semantic rules:*

– *If R is a role name, then $(R^-)^{\mathcal{I}} = \{\langle x, y\rangle : \langle y, x\rangle \in R^{\mathcal{I}}\}$,*
– *If R_1, R_2,..., R_n are roles then $(R_1R_2 \ldots R_n)^{\mathcal{I}} = (R_1)^{\mathcal{I}} \circ (R_2)^{\mathcal{I}} \circ \cdots \circ (R_n)^{\mathcal{I}}$ and $(R_1 \sqcup R_2 \sqcup \ldots \sqcup R_n)^{\mathcal{I}} = (R_1)^{\mathcal{I}} \cup (R_2)^{\mathcal{I}} \cup \cdots \cup (R_n)^{\mathcal{I}}$, where sign $\circ$ is a composition of binary relations,*
– *If C and D are concepts, R is a role, S is a simple role and n is a non-negative integer, then [4]*
  $\top^{\mathcal{I}} = \Delta^{\mathcal{I}}, \bot^{\mathcal{I}} = \emptyset, (\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \backslash C^{\mathcal{I}}, (C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$,
  $(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}, (\exists R.C)^{\mathcal{I}} = \{x \ : \ \exists y. \langle x, y\rangle \in R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\}$,

[4] $\sharp M$ denotes cardinality of set M.

$(\exists S.Self)^{\mathcal{I}} = \{x : \langle x, x\rangle \in S^{\mathcal{I}}\}, (\forall R.C)^{\mathcal{I}} = \{x : \forall y.\, \langle x, y\rangle \in R^{\mathcal{I}} \Rightarrow y \in C^{\mathcal{I}}\}$,
$(\geq nS.C)^{\mathcal{I}} = \{x : \sharp\{y : \langle x, y\rangle \in S^{\mathcal{I}}, y \in C^{\mathcal{I}}\} \geq n\}$,
$(\leq nS.C)^{\mathcal{I}} = \{x : \sharp\{y : \langle x, y\rangle \in S^{\mathcal{I}}, y \in C^{\mathcal{I}}\} \leq n\}$.

Inference problems for $\mathcal{SR}^{\sqcup}\mathcal{IQ}$ are defined in standard way [5].

Definition 6. *An interpretation $\mathcal{I}$ satisfies a RIA $R_1 \cdots R_n \dot{\sqsubseteq} T_1 \sqcup \cdots \sqcup T_m$, if $R_1^{\mathcal{I}} \circ \cdots \circ R_n^{\mathcal{I}} \subseteq T_1^{\mathcal{I}} \cup \cdots \cup T_m^{\mathcal{I}}$. An interpretation $\mathcal{I}$ is model of a*

- *Tbox $\mathcal{T}$ (written $\mathcal{I} \models \mathcal{T}$) if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for each GCI $C \dot{\sqsubseteq} D$ in $\mathcal{T}$.*
- *role hierarchy $\mathcal{R}_h$, if it satisfies all RIAs in $\mathcal{R}_h$ (written $\mathcal{I} \models \mathcal{R}_h$).*
- *role assertions $\mathcal{R}_a$ (written as $\mathcal{I} \models \mathcal{R}_a$) if $\mathcal{I} \models \Phi$ holds for each role assertion axiom $\Phi \in \mathcal{R}_a$, where is $\mathcal{I} \models Dis(S, R)$ if $S^{\mathcal{I}} \cap R^{\mathcal{I}} = \emptyset$,*
  $\mathcal{I} \models Sym(R)$ if $R^{\mathcal{I}}$ is symmetric relation , $\mathcal{I} \models Tra(R)$ if $R^{\mathcal{I}}$ is transitive relation ,
  $\mathcal{I} \models Ref(R)$ if $R^{\mathcal{I}}$ is reflexive relation, $\mathcal{I} \models Irr(S)$ if $R^{\mathcal{I}}$ is irreflexive relation.
- *Rbox $\mathcal{R} = \langle \mathcal{R}_h, \mathcal{R}_a\rangle$ (written as $\mathcal{I} \models \mathcal{R}$) if $\mathcal{I} \models \mathcal{R}_h$ and $\mathcal{I} \models \mathcal{R}_a$.*
- *Abox $\mathcal{A}$ ($\mathcal{I} \models \mathcal{A}$) if for all individual assertions $\phi \in \mathcal{A}$ we have $\mathcal{I} \models \phi$, where*
  $\mathcal{I} \models a : C$ if $a^{\mathcal{I}} \in C^{\mathcal{I}}$, $\mathcal{I} \models a \dot{\neq} b$ if $a^{\mathcal{I}} \neq b^{\mathcal{I}}$,
  $\mathcal{I} \models (a, b) : R$ if $\left\langle a^{\mathcal{I}}, b^{\mathcal{I}}\right\rangle \in R^{\mathcal{I}}$, $\mathcal{I} \models (a, b) : \neg R$ if $\left\langle a^{\mathcal{I}}, b^{\mathcal{I}}\right\rangle \notin R^{\mathcal{I}}$.

For an interpretation $\mathcal{I}$, an element $x \in \Delta^{\mathcal{I}}$ is called an instance of a concept C if $x \in C^{\mathcal{I}}$. An Abox $\mathcal{A}$ is consistent with respect to a Rbox $\mathcal{R}$ and a Tbox $\mathcal{T}$ if there is a model $\mathcal{I}$ for $\mathcal{R}$ and $\mathcal{T}$ such that $\mathcal{I} \models \mathcal{A}$.

Definition 7. *A concept C is called satisfiable if there is an interpretation $\mathcal{I}$ with $C^{\mathcal{I}} \neq \emptyset$. A concept D subsumes a concept C (written $C \dot{\sqsubseteq} D$) if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ holds for each interpretation. Two concepts are equivalent (written $C \equiv D$) if they are mutually subsuming.*

All standard inference problems for $\mathcal{SR}^{\sqcup}\mathcal{IQ}$-concepts and Abox can be reduced [5] to the problem of determining the consistency of a $\mathcal{SR}^{\sqcup}\mathcal{IQ}$-Abox w.r.t. a Rbox, where we can assume w.l.o.g. that all role assertions in the Rbox are of the form $Dis(S, R)$. We call such Rbox reduced.

3 The Extension of $\mathcal{SRIQ}$ Tableau

Let $\mathcal{A}$ be a $\mathcal{SR}^{\sqcup}\mathcal{IQ}$-Abox and $\mathcal{R}$ a reduced $\mathcal{SR}^{\sqcup}\mathcal{IQ}$-Rbox and let $\mathcal{R}_{\mathcal{A}}$ be a set of role names appearing in $\mathcal{A}$ and $\mathcal{R}$, including their inverse, and $\mathcal{I}_{\mathcal{A}}$ is the set of individual names appearing in $\mathcal{A}$. To check whether Abox $\mathcal{A}$ is consistent w.r.t. Rbox $\mathcal{R}$ we transform $\mathcal{SR}^{\sqcup}\mathcal{IQ}$-Rbox $\mathcal{R}$ to $\mathcal{SRIQ}$-Rbox $\mathcal{R}'$ as follows:

1. For each role name $R \in \mathcal{R}_{\mathcal{A}}$ we define equivalence class $[R] = \{R\}$ and set $[R^-] = [R]^-$, $comp([R]) = \{R\}$, $comp([R^-]) = \{R^-\}$,
2. For each RIA of the form $R_{i1} \cdots R_{in_i} \sqsubseteq T_{i1} \sqcup \cdots \sqcup T_{im_i} \in \mathcal{R}$ $(1 \leq i \leq k)$ we define equivalence class $[T_{i1} \sqcup \cdots \sqcup T_{im_i}] = \{T_{j1} \sqcup \cdots \sqcup T_{jm_j} \mid \{T_{i1}, \ldots, T_{im_i}\} = \{T_{j1}, \ldots, T_{jm_j}\}, 1 \leq j \leq k\}$ and set $comp([T_{i1} \sqcup \cdots \sqcup T_{im_i}]) = \{T_{i1}, \ldots, T_{im_i}\}$

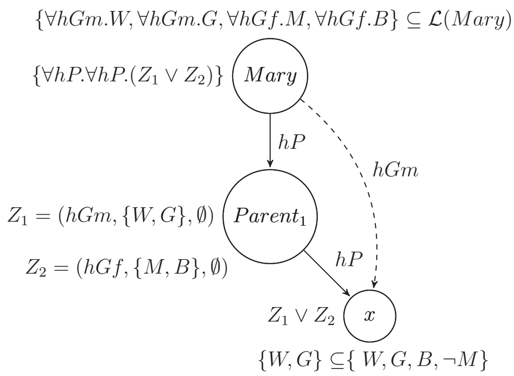

Fig. 1. A part of tableau for (3) and (4)

3. We consider equivalence classes $[R]$, previously defined, as role names which do not appear in $\mathcal{R}_\mathcal{A}$. Set of the role names is denoted with $\mathcal{R}'_\mathcal{A}$. Let's define $\mathcal{R}' = \{[R_1] \cdots [R_n] \dot{\sqsubseteq} [T_1 \sqcup \cdots \sqcup T_m] \mid R_1 \cdots R_n \dot{\sqsubseteq} T_1 \sqcup \cdots \sqcup T_m \in \mathcal{R}\}$.

If Rbox $\mathcal{R}$ is regular w.r.t order $\prec$ then Rbox $\mathcal{R}'$ is regular w.r.t $\prec'$ defined as follows $[R] \prec' [S]$ iff $R \prec S$ and $[T_{ij}] \prec' [T_{i1} \sqcup \cdots \sqcup T_{im_i}]$, $j = 1, ..., m_i$, $i = 1, ..., k$. Equivalence classes and order $\prec'$ previously defined are using for automata construction. For the following example of RIAs $R_1R_2 \sqsubseteq H_1 \sqcup H_2^-$ and $S_1S_2 \sqsubseteq H_2^- \sqcup H_1$ one should construct a nondeterministic finite automaton (NFA) for role $[H_1 \sqcup H_2^-]$. The automaton should accept words R_1R_2 and S_1S_2. Namely, for every role $[R]$ we have kept the construction of NFA $\mathcal{B}_{[R]}$ based on $\mathcal{R}'$, as same as defined in [5]. For $\mathcal{B}$ an NFA and q a state of $\mathcal{B}$, $\mathcal{B}^q$ denotes the NFA obtained from $\mathcal{B}$ by making q the (only) initial state of $\mathcal{B}$ [5]. The language recognized by NFA $\mathcal{B}$ is denoted by $\mathcal{L}(\mathcal{B})$.

To illustrate main idea in this paper, we use the following simple example.

Example 1. In this example we use the following abbreviations: $hP = hasParent, hGm = hasGrandMother, hGf = hasGrandFather, W = Woman, M = Man, G = Gentle, B = Blabber$. We defined the following RIA:

$$hP \circ hP \sqsubseteq hGm \sqcup hGf \tag{3}$$

and the individual assertion:

$$Mary : \forall hGm.W \sqcap \forall hGf.M \sqcap \forall hGm.G \sqcap \forall hGf.B \tag{4}$$

We should decide whether x (see Fig. 1) is instance of $GrandMother$ or $GrandFather$. If $x \in GrandMother^\mathcal{I}$ then $x \in W^\mathcal{I}$, $x \in G^\mathcal{I}$. In the case of $(Mary, x) \in hGm^\mathcal{I}$, it does not break syntax rules. Similar to this one, if $x \in GrandFather^\mathcal{I}$ then $x \in M^\mathcal{I}$, $x \in B^\mathcal{I}$ and $(Mary, x) \in hGf^\mathcal{I}$ hold. Meta-labels Z_1 and Z_2 are using to remember the (relevant) parts of the labels in the node $Mary$ which should be transferred from the node to node x (see Fig. 1). First component in Z_1 is role. The second component is the set of the concepts $\{C|Mary$ is instance of concept $\forall hGm.C\}$. The third component is the set of concepts, for which $Mary$ is instance and should be superset of the set $\{C|x$ is instance of concept $\forall hGm^{-}.C\}$. Because of inverse role we need first and third component. To choose given meta-label, we note as $Z_1 \vee Z_2$. To recognize path $hP \circ hP$ from node $Mary$ to x we use NFA $\mathcal{B}_{[hGm \sqcup hGf]}$ noted as follows $\forall \mathcal{B}_{[hGm \sqcup hGf]}.(Z_1 \vee Z_2)$. $\square$

We assume that all concepts are in negation normal form (NNF). For given concept C_0, $clos(C_0)$ is the smallest set that contains C_0 and that is closed under sub-concepts and $\dot{\neg}$. We use $\dot{\neg}C$ for NNF of $\neg C$ [5]. We use two sets of the label of nodes. First set is [5]: $clos(\mathcal{A}) := \cup_{a:C \in \mathcal{A}} clos(C)$. The second set is:
$NFAclos(\mathcal{A}, \mathcal{R}) := \{\forall \mathcal{B}^q_{[R]}.Z| \ \ [R] \in \mathcal{R}'_{\mathcal{A}}$ and q is state in NFA $\mathcal{B}_{[R]}$ and
$Z = \bigvee_{T \in comp([R])}(T, Z_T, \hat{Z}_T), Z_T \subseteq clos(\mathcal{A})|_T, \hat{Z}_T \subseteq clos(\mathcal{A})|_{T^-}\}$, where
$clos(\mathcal{A})|_Q = \{C \mid \forall Q.C \in clos(\mathcal{A})\}$.

In the proofs of decidability we use set $PL(\mathcal{B}_{[R]}) = \big\{\langle w', q\rangle \,| q$ is a state in $\mathcal{B}_{[R]}$, $(\forall w'' \in \mathcal{L}(\mathcal{B}^q_{[R]}))\big(w'w'' \in \mathcal{L}(\mathcal{B}_{[R]})\big)\big\}$. Set $PL(\mathcal{B}_{[R]})$ contains pairs of the form (w', q). First component w' is prefix of a word $w \in \mathcal{L}(\mathcal{B}_{[R]})$, but the second component q is a state of automaton $\mathcal{B}_{[R]}$ which can be reached if input word for the automaton has prefix w'.

Definition 8. *$T = (\mathbf{S}, \mathcal{L}, \overline{\mathcal{L}}, \mathcal{E}, \mathcal{J})$ is a tableau for $\mathcal{A}$ with respect to $\mathcal{R}$ iff a) $\mathbf{S}$ is non-empty set, b) $\mathcal{L} : \mathbf{S} \to 2^{clos(\mathcal{A})}$, c) $\overline{\mathcal{L}} : \mathbf{S} \to 2^{NFAclos(\mathcal{A},\mathcal{R})}$, d) $\mathcal{J} : \mathcal{I}_{\mathcal{A}} \to \mathbf{S}$, e) $\mathcal{E} : \mathcal{R}_{\mathcal{A}} \to 2^{\mathbf{S}\times\mathbf{S}}$.*
Furthermore, for all $C, C_1, C_2 \in clos(\mathcal{A})$; $s, t \in \mathbf{S}$; $R, S \in \mathcal{R}_{\mathcal{A}}$, and $a, b \in \mathcal{I}_{\mathcal{A}}$, the tableau T satisfies:

- *(P1a) If $C \in \mathcal{L}(s)$, then $\neg\ C \notin \mathcal{L}(s)$ (C is atomic, or $\exists R.Self$),*
- *(P1b) $\top \in \mathcal{L}(s)$, and $\bot \notin \mathcal{L}(s)$, for all s,*
- *(P1c) If $\exists R.Self \in \mathcal{L}(s)$, then $\langle s, s\rangle \in \mathcal{E}(R)$,*
- *(P2) if $(C_1 \sqcap C_2) \in \mathcal{L}(s)$, then $C_1 \in \mathcal{L}(s)$ and $C_2 \in \mathcal{L}(s)$,*
- *(P3) if $(C_1 \sqcup C_2) \in \mathcal{L}(s)$, then $C_1 \in \mathcal{L}(s)$ or $C_2 \in \mathcal{L}(s)$,*
- *(P5) if $\exists S.C \in \mathcal{L}(s)$, then there is some t with $\langle s, t\rangle \in \mathcal{E}(S)$ and $C \in \mathcal{L}(t)$,*
- *(P7) $\langle x, y\rangle \in \mathcal{E}(R)$ iff $\langle y, x\rangle \in \mathcal{E}(Inv(R))$,*
- *(P8) if $(\leq nS.C) \in \mathcal{L}(s)$, then $\sharp S^T(s, C) \leq n$,*
- *(P9) if $(\geq nS.C) \in \mathcal{L}(s)$, then $\sharp S^T(s, C) \geq n$,*
- *(P10) if $(\leq nS.C) \in \mathcal{L}(s)$ and $\langle s, t\rangle \in \mathcal{E}(S)$, then $C \in \mathcal{L}(t)$ or $\dot{\neg}C \in \mathcal{L}(t)$,*
- *(P11) if $a : C \in \mathcal{A}$, then $C \in \mathcal{L}(\mathcal{J}(a))$*
- *(P12) if $(a, b) : R \in \mathcal{A}$, then $(\mathcal{J}(a), \mathcal{J}(b)) \in \mathcal{E}(R)$,*
- *(P13) if $(a, b) : \neg R \in \mathcal{A}$, then $(\mathcal{J}(a), \mathcal{J}(b)) \notin \mathcal{E}(R)$,*
- *(P14) if $a \dot{\neq} b \in \mathcal{A}$, then $\mathcal{J}(a) \neq \mathcal{J}(b)$,*
- *(P15) if $Dis(R, S) \in \mathcal{R}$, then $\mathcal{E}(R) \cap \mathcal{E}(S) = \emptyset$,*
- *(P16) if $\langle s, t\rangle \in \mathcal{E}(R)$ and $R \overset{*}{\sqsubseteq} S$, then $\langle s, t\rangle \in \mathcal{E}(S)$,*[5]

[5] $\overset{*}{\sqsubseteq}$ is the transitive closure of $\sqsubseteq$ [5]

– *(P6') $\forall\mathcal{B}_{[R]}.Z \in \overline{\mathcal{L}}(s)$, where* [6] $Z = \bigvee_{Q\in comp([R])}(Q, Z_Q, \hat{Z}_Q)$, $Z_Q = \mathcal{L}(s)|_Q = \{C|\forall Q.C \in \mathcal{L}(s)\}$ *and* $\hat{Z}_Q = \mathcal{L}(s) \cap clos(\mathcal{A})|_{Q^-}$*, for all* $s \in S$ *and* $[R] \in \mathcal{R}'_{\mathcal{A}}$*,*
– *(P4a') if* $\forall\mathcal{B}^p.Z \in \overline{\mathcal{L}}(s), \langle s,t\rangle \in \mathcal{E}(S)$*, and* $p \xrightarrow{S} q \in \mathcal{B}^p$*, then* $\forall\mathcal{B}^q.Z \in \overline{\mathcal{L}}(t)$*,*
– *(P4b') if* $\forall\mathcal{B}^p.Z \in \overline{\mathcal{L}}(s)$*,* $\varepsilon \in \mathcal{L}(\mathcal{B}^p)$*, and* $Z = \bigvee_{j=1}^{l}(Q_j, Z_j, \hat{Z}_j)$ *then there is* j_0*, such that* $Z_{j_0} \subseteq \mathcal{L}(s)$*,* $\mathcal{L}(s)|_{Q^-_{j_0}} \subseteq \hat{Z}_{j_0}$

where in (P8) and (P9),
$S^T(s,C) = \{t \in \mathbf{S}|\ \langle s,t\rangle \in \mathcal{E}(S')$*, for some* $S' \in \mathcal{L}(\mathcal{B}_S)$ *and* $C \in \mathcal{L}(t)\}\square$.

Lemma 1. *$\mathcal{SR}^{\sqcup}\mathcal{IQ}$-Abox $\mathcal{A}$ is consistent w.r.t. $\mathcal{R}$ iff there exists a tableau for $\mathcal{A}$ w.r.t. $\mathcal{R}$.*

Proof. ($\Leftarrow$)Let $T = (\mathbf{S}, \mathcal{L}, \overline{\mathcal{L}}, \mathcal{E}, \mathcal{J})$ be a tableau for $\mathcal{A}$ with respect to $\mathcal{R}$. An interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ of $\mathcal{A}$ and $\mathcal{R}$ can be defined as follows: $\Delta^{\mathcal{I}} := S$, $C^{\mathcal{I}} := \{s|C \in \mathcal{L}(s)\}$, for a concept name $C \in clos(\mathcal{A})$, $a^{\mathcal{I}} := \mathcal{J}(a)$ for an individual name $a \in \mathcal{I}_{\mathcal{A}}$ and for a role name $[Q] \in \mathcal{R}'_{\mathcal{A}}$, $R \in \mathcal{R}_{\mathcal{A}}$, we set $\overline{\mathcal{E}}([Q]) := \{\langle s_0, s_n\rangle \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}|$ there are $s_1, \cdots, s_{n-1}$ with $\langle s_i, s_{i+1}\rangle \in \mathcal{E}(S_{i+1})$, for $0 \le i \le n-1$ and $S_1S_2\cdots S_n \in \mathcal{L}(\mathcal{B}_{[Q]})\}$, $R^{\mathcal{I}} := \{\langle x,y\rangle \in \cup_{R\in comp([Q])}\overline{\mathcal{E}}([Q])|\mathcal{L}(x)|_R \subseteq \mathcal{L}(y)$ and $\mathcal{L}(y)|_{R^-} \subseteq \mathcal{L}(x)\}$.

We have to show that $\mathcal{I}$ is a model for $\mathcal{A}$ and $\mathcal{R}$.
Next, we show that $\mathcal{I}$ is model for $\mathcal{R}$. $\mathcal{I} \models \mathcal{R}_a$ can be proved by using the same method as in [5]. Let's consider a RIA of the form $R_1\cdots R_n \dot{\sqsubseteq} T_1 \sqcup \cdots \sqcup T_m$. Let's $\langle x_0, x_n\rangle \in (R_1\cdots R_n)^{\mathcal{I}}$. According to semantic rules, there are $x_1, ..., x_{n-1}$ such that $\langle x_i, x_{i+1}\rangle \in R^{\mathcal{I}}_{i+1}$, for $i = 0, 1, ..., n-1$. As roles T_{ij} do not appear on the left hand side of RIAs then $R_i \in comp([Q])$ only for $Q = R_i$ i.e. $R^{\mathcal{I}}_i \subseteq \overline{\mathcal{E}}([R_i])$. This means that there are $y_{i0} = x_i$, y_{i1},...,$y_{il_i} = x_{i+1}$ such that $\langle y_{ij}, y_{ij+1}\rangle \in \mathcal{E}(S_{ij+1})$ and $S_{i1}\cdots S_{il_i} \in \mathcal{L}(\mathcal{B}_{[R_{i+1}]})$. According to automata construction, we have the following: $S_{11}\cdots S_{1l_1}S_{21}\cdots S_{nl_n} \in \mathcal{L}(\mathcal{B}_{[T_1\sqcup\cdots\sqcup T_m]})$ so $\langle x_0, x_n\rangle \in \overline{\mathcal{E}}([T_1\sqcup\cdots\sqcup T_m])$. On the other side, according to rule (P6'), the following $\forall\mathcal{B}_{[T_1\sqcup\cdots\sqcup T_m]}.Z \in \overline{\mathcal{L}}(x_0)$ holds, where $Z = \bigvee_{j=1}^{m}(T_j, Z_{T_j}, \hat{Z}_{T_j})$. By $S_{11}\cdots S_{nl_n} \in \mathcal{L}(\mathcal{B}_{[T_1\sqcup\cdots\sqcup T_m]})$ and rule (P4a') we have $\forall\mathcal{B}^q_{[T_1\sqcup\cdots\sqcup T_m]}.Z \in \overline{\mathcal{L}}(x_n)$ and $\varepsilon \in \mathcal{L}(\mathcal{B}^q_{[T_1\sqcup\cdots\sqcup T_m]})$. From (P4b') we have that there is j such that $\mathcal{L}(x_0)|_{T_j} = Z_{T_j} \subseteq \mathcal{L}(x_n)$ and $\mathcal{L}(x_n)|_{T^-_j} \subseteq \hat{Z}_{T_j} \subseteq \mathcal{L}(x_0)$, i.e. $\langle x_0, x_n\rangle \in T^{\mathcal{I}}_j$. Therefore $\langle x_0, x_n\rangle \in (T_1 \sqcup \cdots \sqcup T_m)^{\mathcal{I}}$.

Secondly, we prove that $\mathcal{I}$ is model for $\mathcal{A}$. We show that $C \in \mathcal{L}(s)$ implies $s \in C^{\mathcal{I}}$ for each $s \in \mathbf{S}$ and each $C \in clos(\mathcal{A})$. Together with (P11)-(P14), this implies that $\mathcal{I}$ is a model for $\mathcal{A}$ [5]. Consider the case $C \equiv \forall R.D$. For the other cases, see [5].
Let $\forall R.D \in \mathcal{L}(s)$ and $\langle s,t\rangle \in R^{\mathcal{I}}$. If R is role name then according to definition $R^{\mathcal{I}}$ there exists $[Q]$ such that $R \in comp([Q])$, $\langle s,t\rangle \in \overline{\mathcal{E}}([Q])$ and $\mathcal{L}(s)|_R \subseteq \mathcal{L}(t)$. If $R = S^-$, where S role name, then according to definition $S^{\mathcal{I}}$ there exists role $[Q]$ such that $S \in comp([Q])$, $\langle t,s\rangle \in \overline{\mathcal{E}}([Q])$ and $\mathcal{L}(s)|_{S^-} \subseteq \mathcal{L}(t)$ (i.e. $\mathcal{L}(s)|_R \subseteq \mathcal{L}(t)$). In both cases we have $D \in \mathcal{L}(t)$. By induction, $t \in D^{\mathcal{I}}$ and thus $s \in (\forall R.D)^{\mathcal{I}}$.

[6] Rules (P6), (P4a) and (P4b) in [5] are changed with rules (P6'), (P4a') and (P4b').

($\Rightarrow$) For the converse, suppose $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ is a model for $\mathcal{A}$ w.r.t. $\mathcal{R}$. We define tableau $T = (\mathbf{S}, \mathcal{L}, \overline{\mathcal{L}}, \mathcal{E}, \mathcal{J})$ as follows:
$\mathbf{S} := \Delta^{\mathcal{I}}$, $\mathcal{J}(a) := a^{\mathcal{I}}$, $\mathcal{E}(R) := R^{\mathcal{I}}$, $\mathcal{L}(s) := \{C \in clos(\mathcal{A})\}|s \in C^{\mathcal{I}}\}$
$\overline{\mathcal{L}}(s) := \{\forall \mathcal{B}^q_{[R]}.Z | (\exists t \in \Delta^{\mathcal{I}})(\exists w')\forall \mathcal{B}_{[R]}.Z \in \overline{\mathcal{L}}_1(t), \langle w', q\rangle \in PL(\mathcal{B}_{[R]})$ and $\langle t, s\rangle \in (w')^{\mathcal{I}}\}$, where $\overline{\mathcal{L}}_1(s) := \{\forall \mathcal{B}_{[R]}.Z | Z = \bigvee_{Q \in comp([R])}(Q, \mathcal{L}(s)|_Q, \mathcal{L}(s) \cap clos(\mathcal{A})|_{Q^-})\}$.

We have to prove that T is tableau for $\mathcal{A}$ w.r.t $\mathcal{R}$. We restrict our attention to the only new cases. For the other cases, see [5].
The rule (P6') follows immediately from the definition of $\overline{\mathcal{L}}_1(s)$ and $\overline{\mathcal{L}}_1(s) \subseteq \overline{\mathcal{L}}(s)$ (for $t = s$ and $w' = \varepsilon$).
For $(P4a')$, let's $\forall \mathcal{B}^p_{[R]}.Z \in \overline{\mathcal{L}}(s), \langle s, t\rangle \in \mathcal{E}(S)$. Assume that there is a transition $p \overset{S}{\rightarrow} q \in \mathcal{B}^p_{[R]}$. From definition $\overline{\mathcal{L}}(s)$ there exists $v \in \Delta^{\mathcal{I}}$ and w' such that $\forall \mathcal{B}_{[R]}.Z \in \overline{\mathcal{L}}_1(v)$, $\langle w', p\rangle \in PL(\mathcal{B}_{[R]})$ and $\langle v, s\rangle \in (w')^{\mathcal{I}}$. Let's $w'' = w'S$ then $\langle w'', q\rangle \in PL(\mathcal{B}_{[R]})$ and $\langle v, t\rangle \in (w'')^{\mathcal{I}}$, so $\forall \mathcal{B}^q_{[R]}.Z \in \overline{\mathcal{L}}(t)$.
For (P4b'), let's $\forall \mathcal{B}^p_{[R]}.Z \in \overline{\mathcal{L}}(s)$, $\varepsilon \in \mathcal{L}(\mathcal{B}^p_{[R]})$, and $Z = \bigvee_{j=1}^{l}(Q_j, Z_j, \hat{Z}_j)$. By definition $\overline{\mathcal{L}}(s)$ there exists $x \in \Delta^{\mathcal{I}}$ and w' such that $\forall \mathcal{B}_{[R]}.Z \in \overline{\mathcal{L}}_1(x), \langle w', q\rangle \in PL(\mathcal{B}_{[R]})$ and $\langle x, s\rangle \in (w')^{\mathcal{I}}$. Further, we have $[R] = [Q_1 \sqcup \cdots \sqcup Q_l]$, $Z_j = \mathcal{L}(x)|_{Q_j}$ and $\hat{Z}_j = \mathcal{L}(x) \cap clos(\mathcal{A})|_{Q_j^-}$. By $\varepsilon \in \mathcal{B}^p_{[R]}$ we have $w' \in \mathcal{L}(\mathcal{B}_{[R]})$, so $w'^{\mathcal{I}} \subseteq (Q_1 \sqcup \cdots \sqcup Q_l)^{\mathcal{I}}$, i.e. $\langle x, s\rangle \in (Q_1 \sqcup \cdots \sqcup Q_l)^{\mathcal{I}}$. This means that there is j such that $\langle x, s\rangle \in Q_j^{\mathcal{I}}$. By the rules of semantics and the definition of $\mathcal{L}(s)$, we have $Z_j = \mathcal{L}(x)|_{Q_j} \subseteq \mathcal{L}(s)$ and $\mathcal{L}(s)|_{Q_j^-} \subseteq \mathcal{L}(x) \cap clos(\mathcal{A})|_{Q_j^-} = \hat{Z}_j$ $\square$.

Tableau algorithm for $\mathcal{SR}^{\sqcup}\mathcal{IQ}$ DL works on the completion forest on similar manner as described in [5].

Definition 9. *(Completion forest) Completion forest for a $\mathcal{SR}^{\sqcup}\mathcal{IQ}$-Abox $\mathcal{A}$ and a Rbox $\mathcal{R}$ is a labeled collection of trees $G = (V, E, \mathcal{L}, \overline{\mathcal{L}}, \dot{\neq})$ whose distinguished root nodes can be connected arbitrarily, where each node $x \in V$ is labeled with two sets $\mathcal{L}(x) \subseteq clos(\mathcal{A})$ and $\overline{\mathcal{L}}(x) \subseteq NFAclos(\mathcal{A}, \mathcal{R})$. Each edge $\langle x, y\rangle \in E$ is labeled with a set $\mathcal{L}(\langle x, y\rangle) \subseteq \mathcal{R}_{\mathcal{A}}$. Additionally, we care of inequalities between nodes in V, of the forest G, with a symmetric binary relation $\dot{\neq}$.*
If $\langle x, y\rangle \in E$, then y is called successor of the x, but x is called predecessor of y. Ancestor is the transitive closure of predecessor, and descendant is the transitive closure of successor. A node y is called an R-successor of a node x if, for some R' with $R' \underline{\sqsubseteq} R$, $R' \in \mathcal{L}(\langle x, y\rangle)$. A node y is called a neighbor (R-neighbor) of a node x if y is a successor (R-successor) of x or if x is a successor (Inv(R))-successor) of y. For $S \in \mathcal{R}_{\mathcal{A}}$, $x \in V$, $C \in clos(\mathcal{A})$ we define set $S^G(x, C) = \{y | y$ is $S-neighbour$ of x and $C \in \mathcal{L}(y)\}$

Definition 10. *A completion forest G is said to contain a clash if there is a node x such that:*

- *$\bot \in \mathcal{L}(x)$, or*
- *for a concept name A, $\{A, \neg A\} \subseteq \mathcal{L}(x)$, or*
- *x is an S-neighbor of x and $\neg\exists S.Self \in \mathcal{L}(x)$, or*

– x *and* y *are root nodes,* y *is an* R*-neighbor of* x*, and* $\neg R \in \mathcal{L}(\langle x, y\rangle)$*, or*
– *there is some* $Dis(R, S) \in \mathcal{R}_a$ *and* y *is an* R *and an* S*-neighbor of* x*, or*
– *there exists a concept* $(\leq nS.C) \in \mathcal{L}(x)$ *and* $\{y_0, \ldots, y_n\} \subseteq S^G(x, C)$ *with* $y_i \dot{\neq} y_j$ *for all* $0 \leq i < j \leq n$*,*
– *there is* $\forall \mathcal{B}^p.Z \in \overline{\mathcal{L}}(x)$*, with* $\varepsilon \in \mathcal{L}(\mathcal{B}^p)$*,* $Z = \bigvee_{j=1}^{l}(Q_j, Z_j, \hat{Z}_j)$ *and there are no* j *such that* $\mathcal{L}(x)|_{Q_j^-} \subseteq \hat{Z}_j$*.*

A completion forest that does not contain a clash is called clash-free. □

The blocking is employed in order to have termination [5].

Definition 11. *A node is called blocked if it is either directly or indirectly blocked [5]. A node* x *is directly blocked if none of its ancestors are blocked, and it has ancestors* x'*,* y *and* y' *such that [5]:*

– *none of* x'*,* y *and* y' *is a root node,*
– x *is a successor of* x' *and* y *is a successor of* y'*, and*
– $\mathcal{L}(x) = \mathcal{L}(y)$ *and* $\mathcal{L}(x') = \mathcal{L}(y')$*, and*
– $\overline{\mathcal{L}}(x) = \overline{\mathcal{L}}(y)$ *and* $\overline{\mathcal{L}}(x') = \overline{\mathcal{L}}(y')$*, and*
– $\mathcal{L}(\langle x', x\rangle) = \mathcal{L}(\langle y', y\rangle)$*.*

In this case we say that y *blocks* x*. A node* y *is indirectly blocked if one of its ancestors is blocked [5].*

The non-deterministic tableau algorithm can be described as follows:

– Input: Non-empty $\mathcal{SR}^{\sqcup}\mathcal{IQ}$-Abox $\mathcal{A}$ and a reduced Rbox $\mathcal{R}$
– Output: "Yes" if $\mathcal{SR}^{\sqcup}\mathcal{IQ}$-Abox $\mathcal{A}$ is consistent w.r.t. Rbox $\mathcal{R}$, otherwise "No"
– Method:
  1. step: Construct completion forest $G = (V, E, \mathcal{L}, \overline{\mathcal{L}}, \dot{\neq})$ as follows:
     - for each individual a occurring in $\mathcal{A}$, V contains a root node x_a,
     - if $(a, b) : R \in \mathcal{A}$ or $(a, b) : \neg R \in \mathcal{A}$, then E contains an edge $\langle x_a, x_b\rangle$,
     - if $a \dot{\neq} b \in \mathcal{A}$, then $x_a \dot{\neq} x_b$ is in G,
     - $\mathcal{L}(x_a) := \{C | a : C \in \mathcal{A}\}$,
     - $\overline{\mathcal{L}}(x_a) := \emptyset$,
     - $\mathcal{L}(\langle x_a, x_b\rangle) := \{R | (a, b) : R \in \mathcal{A}\} \cup \{\neg R | (a, b) : \neg R \in \mathcal{A}\}$

     Go to step 2.
  2. step: Apply an expansion rule (see table 1) to the forest G, while it is possible. Otherwise, go to step 3.
  3. step: If the forest G does not contain *clash* return "Yes", otherwise return "No".

Lemma 2. *Let* $\mathcal{A}$ *be a* $\mathcal{SR}^{\sqcup}\mathcal{IQ}$*-Abox and* $\mathcal{R}$ *a reduced Rbox. The tableau algorithm terminates when started for* $\mathcal{A}$ *and* $\mathcal{R}$*.*

Lemma 3. *Let* $\mathcal{A}$ *be a* $\mathcal{SR}^{\sqcup}\mathcal{IQ}$*-Abox and* $\mathcal{R}$ *a reduced Rbox. Tableau algorithm returns answer "Yes" if and only if there is a tableau for* $\mathcal{A}$ *w.r.t.* $\mathcal{R}$*.*

Table 1. Expansion rules for $\mathcal{SR}^{\sqcup}\mathcal{IQ}$ tableau algorithm (updated from [5])

	The rules $\sqcap$, $\sqcup$, $\exists$, Self, $\leq_r$, $\geq$, $\leq$ are defined in [5], but only in rules that create new node y should set $\overline{\mathcal{L}}(y) := \emptyset$
ch'	If x is not indirectly blocked and there is concept $C \in clos(\mathcal{A})$ with $\{C, \dot{\neg} C\} \cap \mathcal{L}(x) = \emptyset$ then $\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{E\}$, for some $E \in \{C, \dot{\neg} C\}$
$\forall'_1$	If x is not indirectly blocked and it is not possible to apply ch'-rule to $\mathcal{L}(x)$, and $\forall \mathcal{B}_{[R]}.Z \notin \overline{\mathcal{L}}(x)$, where $Z = \bigvee_{Q \in comp([R])} (Q, \mathcal{L}(x)\|_Q, \mathcal{L}(x) \cap clos(\mathcal{A})\|_{Q^-})$ then $\overline{\mathcal{L}}(x) \rightarrow \overline{\mathcal{L}}(x) \cup \{\forall \mathcal{B}_{[R]}.Z\}$
$\forall'_2$	If $\forall \mathcal{B}^p.Z \in \overline{\mathcal{L}}(x)$, and x is not indirectly blocked, $p \xrightarrow{S} q \in \mathcal{B}^p$ and there is S-neighbor y of x with $\forall \mathcal{B}^q.Z \notin \overline{\mathcal{L}}(y)$ then $\overline{\mathcal{L}}(y) \rightarrow \overline{\mathcal{L}}(y) \cup$ $\{\forall \mathcal{B}^q.Z\}$
$\forall'_3$	If $\forall \mathcal{B}^p.Z \in \overline{\mathcal{L}}(y)$, and y is not indirectly blocked, $\varepsilon \in \mathcal{L}(\mathcal{B}^p)$, $Z = \bigvee_{j=1}^{l} (Q_j, Z_j, \hat{Z}_j)$ and there is no j such that $Z_j \subseteq \mathcal{L}(y)$ and $\mathcal{L}(y)\|_{Q_j^-} \subseteq \hat{Z}_j$ then choose j such that $\mathcal{L}(y)\|_{Q_j^-} \subseteq \hat{Z}_j$ and $\overline{\mathcal{L}}(y) \rightarrow \overline{\mathcal{L}}(y) \cup Z_j$.

Proof. For the if direction, suppose that the algorithm returns "Yes". It means that the algorithm generated completion forest $G = (V, E, \mathcal{L}, \overline{\mathcal{L}}, \dot{\neq})$ without clash and there are no expansion rules (see table 1) that can be applied.

Let's $b(x) = x$, if x is not blocked and $b(x) = y$, if y blocks node x.

A path [6] is a sequence of pairs nodes of $\mathbf{G}$ of the form

$$p = \langle (x_0, x'_0), \ldots, (x_n, x'_n) \rangle \,. \tag{5}$$

For such a path, we define $Tail(p) = x_n$ and $Tail'(p) = x'_n$. We denote the path

$$\left\langle (x_0, x'_0), (x_1, x'_1), \ldots, (x_n, x'_n), (x_{n+1}, x'_{n+1}) \right\rangle \tag{6}$$

with $\left\langle p \middle| (x_{n+1}, x'_{n+1}) \right\rangle$. The set of $Paths(\mathbf{G})$ can be defined inductively as follows:

- if x_0 is root node then $\langle x_0, x_0 \rangle \in Paths(\mathbf{G})$
- if $p \in Paths(\mathbf{G})$, $z \in V$ and z is not indirectly blocked, such that $\langle Tail(p), z \rangle \in E$, then $(p, \langle b(z), z \rangle) \in Paths(\mathbf{G})$

We define structure $T = (\mathbf{S}, \mathcal{L}, \overline{\mathcal{L}}, \mathcal{E}, \mathcal{J})$ as follows $\mathbf{S} := Paths(\mathbf{G})$, $\mathcal{L}(p) := \mathcal{L}(Tail(p))$, $\overline{\mathcal{L}}(p) := \overline{\mathcal{L}}(Tail(p))$, if root node x_a denotes individual a then $\mathcal{J}(a) = (\langle x_a, x_a \rangle)$ and $\mathcal{E}(R) := \{\langle s, t \rangle \in \mathbf{S} \times \mathbf{S} | t = (s, \langle b(y), y \rangle)$ and y is an $R-$successor of $Tail(s)$ or $s = (t, \langle b(y), y \rangle)$ and y is an $Inv(R)-$successor of $Tail(t)\}$ $\cup \{\langle \mathcal{J}(a), \mathcal{J}(b) \rangle \, | x_b$ is an R-neighbour of $x_a\}$.

Thus defined structure T is a tableau. New rules (P6'), (P4a') directly follows from $\forall'_1$ and $\forall'_2$ rule, but (P4b') follows from $\forall'_3$ and definition of clash (see definition (10)). For the other cases, see [6].

For the only-if direction, the proof is the same as proof in [4, 5] (i.e., we take a tableau and use it to steer the application of the non-deterministic rules).$\Box$

From Theorem 1 in [5] and Lemmas 1, 2 and 3, we thus have the following theorem:

Theorem 1. *The tableau algorithm decides satisfiability and subsumption of $\mathcal{SR}^{\sqcup}\mathcal{IQ}$-concepts with respect to Aboxes, Rboxes, and Tboxes.*

4 Conclusion

It is important to note that original idea of extension $\mathcal{ALC}$ DL with composition-based RIAs is presented in [11]. We introduce more expressive formalism that allows composition-based RIAs and relaxed restrictions defined in [11]. Motivated by practical applications in manufacturing engineering we define tableau algorithm in order to check satisfiability of $\mathcal{SR}^{\sqcup}\mathcal{IQ}$ DL. Future research will be focused on how to extend regularity conditions for $\mathcal{SROIQ}$ DL in order to support composition-based RIAs as well as at the same time support RIAs proposed in [9]. We use the algorithm proposed in this paper for modeling the regulations of capital adequacy of credit institutions.

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