Spatial Reasoning for Image Retrieval

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Abstract

Building upon the description logic of concrete domains introduced by Baader and Hanschke, we provide a system which is aimed at enhancing image retrieval with the ability to perform spatial reasoning. In this early stage of our research we focus on obtaining a formalism which is expressive enough to increase the usefulness of expensive picture descriptions (common in content based image retrieval systems) by means of spatial reasoning.

1 Introduction

With the continuing advances in multi-media technology, the amount of information at our disposal is not only increasing in quantity but also in variety. Nowadays, retrieval mechanisms should be able to deal both with textual information, and with sounds, images, videos, etc. When trying to retrieve a picture, sentences like "I want to see a picture of a *person inside a booth*" come naturally. From this sentence we can infer that in any picture correctly described by the portion in italics, both a person and a booth should appear, but also that certain spatial relations should hold between them. Suppose we query an image database so that all pictures conforming to this description are retrieved. Intuitively, the result from such a query and the result obtained from the description "booth containing a person" should coincide. For this to be possible in full generality, the system should be able to perform spatial inferences on the objects described in the picture.

We will provide a formalism for picture description and querying which is built upon the description logic of concrete domains $\mathcal{ALC}(\mathcal{D})$ introduced in [1]. As our specific concrete domain (where spatial reasoning will take place) we choose the RCC8 calculus [3]. Picture descriptions will materialize as knowledge bases over $\mathcal{ALC}(\mathcal{D}_{\text{rcc8}})$, and picture selection will be implemented as inference. By results in [10], the retrieval process we propose will be a reasoning task in PSPACE. Given that reasoning takes place in a fairly complex framework, this is the optimal "worst case complexity" we can hope for. Empirical tests with other widely used description logics with reasoning tasks also in PSPACE are encouraging, though, as several implementations show extremely good performance [8].

The paper is organized as follows. In Section 2 we discuss basic background material on Image Retrieval Systems (IRSs). In Section 3 we introduce a description logic to provide picture descriptions, we comment on the retrieval mechanism and provide an example of spatial reasoning at the concrete level. In Section 4 we briefly review related work and in Section 5 we draw our conclusions.

2 Background

Most IRSs in use today are based only on the syntactic content of the image, i.e., only physical properties are considered (like color distribution, illumination and intensity). The semantic content of the image is considered in only very few cases.

An example of an implementation of an IRS is the Altavista AVPhoto photograph search engine (http://image.altavista.com). If we present AVP with our example query "person inside a booth" we are correctly answered with a picture of a woman inside a British red telephone booth. But if we query for a "booth containing a person" we don't get the same photograph as before. Furthermore, if we ask for photographs similar to the one correctly found the first time, we simply get pictures whose main color is red, like our British telephone booth, but that we wouldn't call similar at all, such as a picture of Santa Claus or of the cover page of a CD-Rom war game.

More generally, based on the syntax-semantics dichotomy we can distinguish two types of IRSs. In the first, syntactic type, images are stored according to some physical value of the image. A query is an image itself and the result is an image in the database whose "distance" to the query is the smallest. State of the art examples in the field are [4, 5]. The main advantage resides in the feasibility of the database; large collections of elements from different domains can automatically be indexed at low costs. Nonetheless, retrieval based only on the syntactic features of images is very poor and unsatisfactory for most applications. In contrast, content-based approaches working on semantic descriptions of the images can perform accurate and satisfactory searches. The price is that images must be pre-processed (at least partially) by a human operator in order to obtain an image description. This kind of IRSs usually applies to small databases, with little variety in domain, that are quite costly and time consuming to implement.

Where can we improve? The ultimate goal is to have a system that can automatically process an image, extracting syntactic information. It will then use this data, eventually combined with general domain information, to extract semantic content and, finally, store both types of information together with the original image. This is still way out of reach; our aim in this article is to *maximize the usefulness* of semantic image descriptions that are currently available by introducing spatial reasoning.

3 The Formalism

In this section we present the basic notions concerning $\mathcal{ALC}(\mathcal{D})$ (see [1] for further details); we define $\mathcal{ALC}(\mathcal{D}_{\text{rcc8}})$ and present an example of concrete spatial reasoning.

Definition 1 A *concrete domain* $\mathcal{D}$ is a non-empty set $\text{dom}(\mathcal{D})$ (the domain) and a set $\text{pred}(\mathcal{D})$, the predicate names of $\mathcal{D}$. Each n-ary predicate name P is associated with an n-ary relation in $\text{dom}(\mathcal{D})$. A concrete domain $\mathcal{D}$ is *admissible* if the set of its predicate names is closed under negation and contains a name for $\text{dom}(\mathcal{D})$, and the satisfiability problem for finite conjunctions of predicates is decidable.

Selecting an appropriate concrete domain is non-trivial. We should achieve a compromise between an expressive formalism which would be useful for picture descriptions, while still remaining admissible. The RCC8 calculus [3] over the domain of all non-empty regular closed subset of $\mathbb{R}^2$ seems to be a good candidate [9].

Definition 2 We define the concrete domain $\mathcal{D}_{\text{rcc8}}$ such that $\text{dom}(\mathcal{D}_{\text{rcc8}})$ is the set of all non-empty regular closed subsets of $\mathbb{R}^2$ and $\text{pred}(\mathcal{D}_{\text{rcc8}})$ is obtained by union, intersection, composition and converse over the set $\{PO$, $NTPP$, TPP, EQ, TPP^{-1}, $NTPP^{-1}$, EC, DC, $U\}$ with the intended meaning of Proper Overlap, Non Tangential Proper Part, Tangential Proper Part, EQual, Converse Tangential Proper Part, Converse Non Tangential Proper Part, External Connection, DisConnected, and Universal, respectively.

Deciding satisfaction for RCC8 is NP-complete [12]. Furthermore, RCC8 relations are closed under boolean operations. Hence $\mathcal{D}_{\text{rcc8}}$ is admissible. Given the NP results for RCC8, deciding satisfiability in $\mathcal{ALC}(\mathcal{D}_{\text{rcc8}})$ (with empty T-boxes) is PSPACE [10].

Definition 3 ($\mathcal{ALC}(\mathcal{D}_{\text{rcc8}})$) Fix a signature $\tau = \langle \mathsf{C}, \mathsf{R}, \mathsf{F}, \mathsf{CN}, \mathsf{AN}\rangle$ (of Atomic Concepts, Roles, Features, Concrete Names and Abstract Names, respectively) where each component is a non-empty set and they are all pairwise disjoint. Then, the sets CON of *concepts*, TERM of *terminological axioms* and ASSER of *assertional axioms* are defined as

$$\text{CON} := C \mid \neg\text{CON} \mid \text{CON} \sqcap \text{CON} \mid \forall R.\text{CON} \mid P(u_1, \ldots, u_n)$$
$$\text{TERM} := C \sqsubseteq D$$
$$\text{ASSER} := a{:}\,C \mid (a,b){:}\,R \mid (a,b){:}\,f \mid (a,y){:}\,f \mid (y_1,\ldots,y_n){:}\,P$$

where $C \in \mathsf{C}$, $D \in \text{CON}$, $R \in \mathsf{R}$, $P \in \text{pred}(\mathcal{D}_{\text{rcc8}})$ n-ary, $u_1, \ldots, u_n \in \mathsf{F}^+$, $f \in \mathsf{F}$, $a, b \in \mathsf{AN}$ and $y, y_1, \ldots, y_n \in \mathsf{CN}$.

A *knowledge base* K is a tuple $\langle T, A\rangle$ where $T \subseteq$ TERM and $A \subseteq$ ASSER; furthermore, we require T to be acyclic.

An *interpretation* $\mathcal{I}$ is a tuple $\langle \Delta, \cdot^{\mathcal{I}}\rangle$ where Δ is a non-empty set disjoint from $\text{dom}(\mathcal{D}_{\text{rcc8}})$ and $\cdot^{\mathcal{I}}$ maps elements of C to subsets of Δ, elements of R to subsets of $\Delta \times \Delta$, elements of F to partial functions from Δ into $\Delta \cup \text{dom}(\mathcal{D}_{\text{rcc8}})$, elements of CN to elements of $\text{dom}(\mathcal{D}_{\text{rcc8}})$, and elements of AN to elements of Δ. The interpretation of elements in F can be extended to elements of F^+ by taking compositions, and to elements of $(\mathsf{CN} \cup \mathsf{AN})^n$ as $(v_1, \ldots, v_n)^{\mathcal{I}} = (v_1^{\mathcal{I}}, \ldots, v_n^{\mathcal{I}})$. Furthermore, $\mathcal{I}$ can be extended to formulas in CON as follows: $(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$, $(\neg I)^{\mathcal{I}} = \Delta \setminus C^{\mathcal{I}}$, $(\forall R.C)^{\mathcal{I}} = \{d \in \Delta \mid \forall d'.(d,d') \in R^{\mathcal{I}} \Rightarrow d' \in C^{\mathcal{I}}\}$, and $(P(u_1, \ldots, u_n))^{\mathcal{I}} = \{d \in \Delta \mid \exists r_1, \ldots, r_n \in \text{dom}(\mathcal{D}_{\text{rcc8}}).\ u_1^{\mathcal{I}}(d) = r_1, \ldots, u_n^{\mathcal{I}}(d) = r_n \;\&\; (r_1, \ldots, r_n) \in P\}$.

Define a satisfaction relation $\models$ between interpretations and elements of TERM $\cup$ ASSER as follows. $\mathcal{I} \models C \subseteq D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$; and $\mathcal{I} \models \bar{b}{:}\,A$ iff $\bar{b}^{\mathcal{I}} \in A^{\mathcal{I}}$. This notion trivially extends to subsets of TERM $\cup$ ASSER.

Finally, given knowledge bases $K = \langle T, A\rangle$ and $K' = \langle T', A'\rangle$, we say that $K \models K'$ iff for all interpretations $\mathcal{I}$, $\mathcal{I} \models T \cup A$ implies $\mathcal{I} \models T' \cup A'$.

The language $\mathcal{ALCRP}(\mathcal{D})$ was introduced in [6]. $\mathcal{ALCRP}(\mathcal{D})$ is the extension of $\mathcal{ALC}(\mathcal{D})$ with the role forming operator $P(\bar{u})(\bar{v})$ whose interpretation is defined as $(P(u_1, \ldots, u_n)(v_1, \ldots, v_m))^{\mathcal{I}} = \{(a,b) \in \Delta^{\mathcal{I}} \mid \exists \bar{x}, \bar{y} \in \text{dom}(\mathcal{D}_{\text{rcc8}})$ such that $u_i^{\mathcal{I}}(a) = x_i, v_i^{\mathcal{I}}(b) = y_i$ and $(x_1, \ldots, x_n, y_1, \ldots, y_m) \in P\}$. $\mathcal{ALCRP}(\mathcal{D}_{\text{rcc8}})$ is related to $\mathcal{ALCRP}(S2)$ introduced in [7].

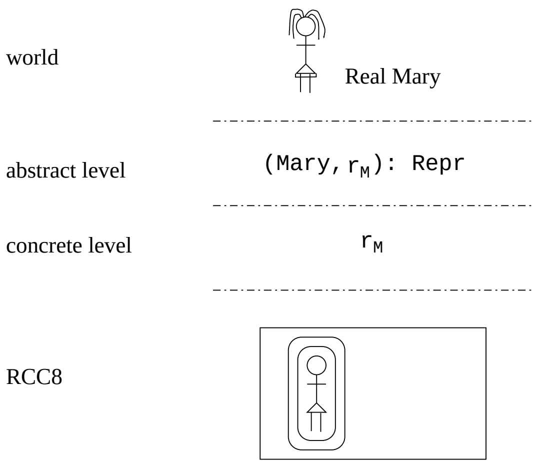

Figure 1: The different representation levels.

Notice that description logics of concrete domains let us treat information at different levels. Figure 1 gives some examples.

We are now ready to give our definition of an image database and of the retrieval mechanism.

Definition 4 Given a finite set of pictures p_i, an *image database* ID consists of a finite set of knowledge bases $ID = \{K_1, \ldots, K_n\}$, where each K_i corresponds to the description of p_i. A *query* φ is a knowledge base: $\varphi \subseteq \text{TERM} \cup \text{ASSER}$. Given a query φ, the *retrieval process* will return the set $\text{Retr}(\varphi) = \{p_i \mid \langle T_i, A_i \rangle \models \varphi,\ \text{for } K_i = \langle T_i, A_i \rangle \in ID\}$. When domain information is present, it can be encoded as a background knowledge base $KB = \langle T, A \rangle$. Then $\text{Retr}(\varphi)$ is defined as $\{p_i \mid \langle T \cup T_i, A \cup A_i \rangle \models \varphi,\ \text{for } K_i = \langle T_i, A_i \rangle \in ID\}$.

As retrieval is modeled as inference it can be performed in PSPACE over the unfolded knowledge box.

It is important to notice that an image database is actually a *collection* of knowledge bases. We can think of the concrete domain $\mathcal{D}_{\text{rcc8}}$ as a surface where the image in each knowledge base is projected. If we join all the descriptions in a unique database, we would be superimposing the images. Also, the architecture of the image database (a set of small descriptions of pictures plus a shared background knowledge base) reflects the different kinds of information stored, see Figure 2. Specific information about the picture is stored "locally," while general facts about the domain are available globally, thus avoiding redundancies.

3.1 Using the Formalism

$\mathcal{ALC}(\mathcal{D}_{\text{rcc8}})$ is an expressive formalism for the description of images. Below we discuss some of the steps which will commonly be carried out in a description of a picture referring to the following examples.

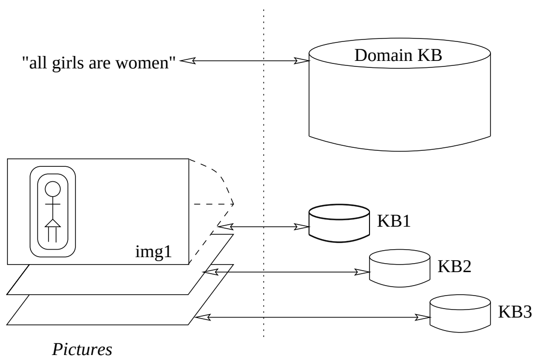

Figure 2: The architecture of the system.

(1)	$\texttt{Mary}\colon\texttt{woman}$
	$\texttt{BBooth}\colon\texttt{telephone{-}booth}$
(2)	$(\texttt{Mary}, \texttt{r}_M)\colon\texttt{Repr}$
	$(\texttt{BBooth}, \texttt{r}_B)\colon\texttt{Repr}$
	$(\texttt{r}_M, \texttt{r}_B)\colon \mathit{PP}$
(3)	$\texttt{woman} \sqsubseteq \texttt{female} \sqcap \texttt{human}$
(4)	$\texttt{Mary}\colon \exists\texttt{WEARS.red}$
(5)	$(\texttt{Mary}, \texttt{Peter})\colon\texttt{TO{-}RIGHT}$
(6)	$\texttt{Mary}\colon \mathit{Universal}(\texttt{Repr})$
(7)	$\neg((\texttt{woman} \sqcap \mathit{Universal}(\texttt{Repr})) \doteq \bot)$
(8)	$(\texttt{Mary}, \texttt{Peter})\colon \mathit{PO}$
(9)	$(\texttt{Mary}, \texttt{Peter})\colon \mathit{PO}(\texttt{Repr})(\texttt{Repr})$
(10)	$(\texttt{Mary}, \texttt{r}_P)\colon\texttt{ReprPeter}$
(11)	$\texttt{Mary}\colon \mathit{PO}(\texttt{Repr}, \texttt{ReprPeter})$

1. Linking abstract names to concrete regions. This is the first fundamental task. Most of the knowledge that does not correspond to physical aspects of the picture will refer to abstract elements which do not appear 'per se' in the picture. For example, in describing a picture of `Mary` we want to say not only that `Mary` is a woman and `BBooth` is a telephone booth (1), but also that they stand in certain spatial relations in the picture: the region corresponding to `Mary` is a proper part (*PP*) of the region corresponding to `BBooth`. To this end, we select a feature `Repr` wich will be used to link abstract names (like `Mary`) with their concrete regions ($\texttt{r}_M$) as in (2).

2. Adding abstract information. Information about `Mary` can now be added without particular difficulties in the standard "description logic" way. But it should be a responsibility of the designer of the knowledge base to decide which information pertains to the picture and which is domain information. The fact that women are female humans (3) does not correspond to the description of the picture, while (4) "Mary wears red clothes" does. Notice that some kinds of spatial information (which can not be handled by RCC8) can be specified at this level. For example, we can explicitly say that `Mary` is to the right of `Peter` (5), but further knowledge is needed to infer that then `Peter` is to the left of `Mary`; no reasoning apart from look up in the knowledge base will be available for such predicates.

3. Handling queries. This is best explained by examples. Some queries are easy to specify. (6) asks for all the pictures where `Mary` appears. Queries like "all pictures showing a woman" are of a different kind. We could write this query as (7), but this involves a complex axiom for which the complexity results in [10] would not hold. We can overcome this problem, by rewriting the query in terms of A-box assertions: pick a new constant $\mathtt{x}$ not present in the knowledge base K, then $K \models \mathtt{x}\colon\neg C$ iff $K \models C \doteq \bot$. Hence, we could answer the query "all pictures showing a woman" as the set of knowledge bases $\{K \mid K \not\models \mathtt{x}\colon \neg(\mathtt{woman} \sqcap \mathit{Universal}(\mathtt{Repr})\}$. Finally, suppose we would like to query for those pictures where the image of `Mary` covers the image of `Peter`. This is more complex than saying (8), as overlapping is a relation holding between *regions*. The natural way to express this query would be (9), but this requires the "relation lifting" operation of $\mathcal{ALCRP}(\mathcal{D}_{\mathrm{rcc8}})$. Unrestricted use of relational lifting quickly leads to undecidability, but in our case it is needed in a very restricted way: if the description of the picture contains a constant feature `ReprPeter` which always returns the region associated to `Peter` (see (10)), we can express the query in $\mathcal{ALC}(\mathcal{D}_{\mathrm{rcc8}})$ as (11). We will comment further on this in the next subsection.

An attractive characteristic of the framework we propose is that queries and picture descriptions are at the same level. In the same way as a syntax-oriented image database is queried via a syntactic element (a picture), our semantics-oriented database is queried through a description. This analogy naturally leads to transfer of ideas. In [5] a method to index images, resulting in a bi-dimensional graph layout encoding similarities, is proposed. Distances on this graph are then used to improve retrieval. In our framework relations of similarity would be subsumption or partial subsumption between image descriptions which can be computed off-line and used as guidance during queries [2].

3.2 Spatial Reasoning in Action

In the previous subsection we provided general remarks concerning picture descriptions in the image database. Let us now consider which formalizations and spatial inferences are possible in our system. Reconsider the British phone booth example presented in the introduction. Issues related to how elements are represented are obviously very relevant. Suppose we consider both `Mary` and `BBooth` to be abstract names. Then describing a picture representing "Mary inside the red British booth" is not problematic.

"Mary inside the red British booth"
$(\mathtt{Mary}, \mathtt{r}_M)\colon \mathtt{Repr}$
$(\mathtt{Mary}, \mathtt{r}_B)\colon \mathtt{ReprBBooth}$
$\mathtt{Mary}\colon \mathtt{woman}$
$(\mathtt{BBooth}, \mathtt{r}_B)\colon \mathtt{Repr}$
$(\mathtt{BBooth}, \mathtt{r}_M)\colon \mathtt{ReprMary}$
$\mathtt{BBooth}\colon \mathtt{red}\ (\mathtt{r}_M, \mathtt{r}_B)\colon PP$

Furthermore, the description above implies (i.e., will be retrieved by) both the queries "Booth containing Mary" ($\mathtt{BBooth}\colon PP^{-1}(\mathtt{Repr}, \mathtt{ReprMary})$) and "Mary inside the Booth" ($\mathtt{Mary}\colon PP(\mathtt{Repr}, \mathtt{ReprBBooth})$).

Now consider the query "Person inside the British Booth." Intuitively, `person` should be represented as a concept, not as an abstract name. The query should then be written as $\neg(\mathtt{person} \sqcap PP(\mathtt{Repr}, \mathtt{ReprBBooth}) \doteq \bot)$, which as we described in Section 3.1, item 2, can be formulated in terms of assertions. Assuming the background knowledge $\mathtt{woman} \sqsubseteq \mathtt{human} \sqcap \mathtt{female}$ and $\mathtt{human} \sqsubseteq \mathtt{person}$, the picture of Mary will be retrieved. On the other hand, the question "Mary inside something red" poses a problem. It cannot be directly written in the formalism. Only the equivalent "Something red containing Mary" can be expressed via the use of constant features. Given the fact that all relations in $\mathrm{pred}(\mathcal{D}_{\mathrm{rcc8}})$ have converses, this translation can always be performed.

As a final example, suppose that `Peter` was also in the picture, standing outside the telephone booth. We can represent this by $(\mathtt{Peter}, \mathtt{r}_P) : \mathtt{Repr}$ and $(\mathtt{r}_P, \mathtt{r}_B)\colon DC$ specifying his relative position with the booth. This picture would be retrieved by the query "Pictures where `Mary` and `Peter` do not overlap."

To sum up, $\mathcal{ALC}(\mathcal{D}_{\mathrm{rcc8}})$ can express spatial relations between abstract names, and between abstract names and concepts but not between two concepts. The additional expressive power introduced by the role forming operator is needed in the last case. "Something red containing a person" would be written $\mathtt{red} \sqcap \exists PP^{-1}(\mathtt{Repr})(\mathtt{Repr}).\mathtt{person}$. Notice that this kind of queries still falls inside a decidable fragment of $\mathcal{ALCRP}(\mathcal{D}_{\mathrm{rcc8}})$ but no complexity result is available yet for this language (see [6]).

4 Related Work

Our proposal is related to the two-level formalisms presented in [11] but there are two important differences. We change the basic fuzzy set-up for a standard two-valued logic at the abstract level, and we enhance this abstract level with the ability to perform spatial inferences by using the description logic $\mathcal{ALC}(\mathcal{D}_{\mathrm{rcc8}})$. In [11] an underlying layout associated to each image is defined. In that setting the layout is a triple: a matrix of points (pixels of the image), a partition of these pixels into atomic regions and a function associating a color to each region. A constraint states that two touching regions cannot have the same colors. Furthermore, there is no distinction between open and closed regions. This kind of setting is enough to describe many situations, but in general overlapping regions may have the same color.

5 Conclusions and Future Work

We have introduced a description logic built on top of concrete domains as a basis for content based Image Retrieval Systems. The novelty resides in the explicit possibility to perform spatial reasoning on the entities in the pictures. Drawing on results in [10] we have also established the complexity of our formalism and analyzed the expressive power involved in various kinds of spatial queries. Description logics based on concrete domains seem well-suited for this purpose and constitute an elegant and flexible example of combined decision methods (a tableau system is used at the "logic" side of the reasoning task, while specific methods solve spatial constraints).

There exist many directions for further research. In this paper we are mainly concerned with A-Box reasoning; understanding T-Box spatial reasoning is indeed interesting. Given that description logics are a powerful aid to build hierarchies, our approach can be applied as a tool for organizing pictures into categories [2]. Restricting the domain of application is another possibility. For example, all images might correspond to city outdoors. In this case, an important amount of relevant spatial information on cities would be available. Images would be treated according to some image processing technique. Then, the information gathered is matched against the database to try to attach semantic content to syntactic elements. This can be done at two levels: from the lines and regions identified in the image, or after grouping these lines and regions in semantically coherent clusters (e.g., three parallelograms in external contact could be grouped into a box). The subsumption in the logic could then be performed on information extracted from images, previously present information (city knowledge), and, eventually, low level information about the images.

The ideas introduced in this paper may provide the foundations for an interactive approach to IRSs. In a first stage, the system will detect lines automatically (by image processing algorithms). The user provides assistance for grouping lines into region's boundaries. The system then uses domain related knowledge on shapes and objects to refine the boundaries defining regions, iteratively. When a final layout of regions is found, the user attaches semantic content to them: "Yes, this is `Mary`" when presented the region *mary*. The system then stores this information as part of the description of the picture.

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