On the Semantics of Linking and Importing in Modular Ontologies *

Jie Bao, Doina Caragea and Vasant Honavar
Artificial Intelligence Research Laboratory
Computer Science Department
Iowa State University, Ames, IA USA 50010
Email: {baojie, dcaragea, honavar}@cs.iastate.edu

1 Introduction

Because the web is a network of loosely coupled, distributed, autonomous entities, it is inevitable that the ontologies on the web to be modular, collaboratively built and partially connected. However, the current web ontology language OWL provides only limited support for modular ontologies. Consequently, it fails to accomodate localized semantics, partial reuse, selective knowledge hiding, and scalable inference [2]. Hence, there is significant interest on modular ontology languages, such as Fusion of Abstract Description Systems (FADS) [1], Distributed Description Logics (DDL)[3], $\mathcal{E}$-connections [8; 6] and Package-based Description Logics (P-DL) [2].

These proposals adopt two broad classes of approaches to asserting and using semantic relations between multiple ontology modules: use of *m*appings or *linkings* between ontology modules e.g., DDL and to a large extent, $\mathcal{E}$-connections; and the use of *i*mporting e.g., P-DL. The major difference between the two approaches has to do with the use of "foreign terms" in ontology modules. In a linked ontology, different modules have *disjoint terminologies* and *disjoint interpretation* domains, and semantic relations between ontology modules are only enabled by a set of *mapping axioms*, such as bridge rules in DDL or $\mathcal{E}$-connections. In contrast, *importing* allows an ontology module to make direct reference to terms defined in other ontology modules, i.e., *importing of foreign terms*.

Serafini *et.al.* (2005) [10] compare mapping or linking based approaches to "integration" of multiple ontology modules such as DDL and $\mathcal{E}$-connections by reducing them to the Distributed First Order Logics (DFOL) [4] framework. However, there is little work on formal investigation of the importing approach to integrating ontology modules. Against this background, we compare the semantics of linking in DDL, $\mathcal{E}$-connections, and importing in P-DL within the DFOL framework.

2 Desiderata For Modular Ontologies

We first list a set of minimal requirements for modular ontologies on the semantic web as the basis for our comparison of the semantics of DDL, $\mathcal{E}$-connections and P-DL within the DFOL framework:

*This research is supported by grants from NSF (0219699) and NIH (GM 066387). A longer version is available at `http://archives.cs.iastate.edu/documents/disk0/00/00/04/08/index.html`

1. **Localized Semantics**. A modular ontology should not only be *syntactically modular* (e.g. stored in separated XML name spaces), but also *semantically modular*. That is, the existence of a *global model* should not be a requirement for integration of ontology modules.
2. **Exact Reasoning**. The answer to a reasoning problem over a collection of ontology modules should be *semantically equivalent* to that obtained by reasoning over an ontology resulting from an appropriate *integration* of the relevant ontology modules.
3. **Support for Directional Semantic Relations**. The framework must support *directional semantic relations* from a *source* module to a *target* module. A directional semantic relation affects only the reasoning within the target module and not the source module.
4. **Transitive Reusability**. Knowledge contained in ontology modules should be directly or indirectly reusable. That is, if a module A reuses modules B and C, and module C reuses modules D and E, then effectively, module A reuses modules D and E.

Other desiderata that have been considered in the literature include: the ability to cope with local inconsistency or global inconsistency, and local logic completeness. We believe that ones listed above are among the most critical ones for a modular ontology to be semantically sound and practically usable.

3 Distributed First Order Logics

A DFOL (Ghidini and Serafini,[4]) knowledge base (KB) (and hence, a DFOL ontology) includes a family of first order languages $\{L_i\}_{i\in I}$, defined over a finite set of indices I. We will use L_i to refer to the ith module of the ontology . An (i-)variable x or (i-)formula ϕ occurring in module L_i is denoted as $i : x$ or $i : \phi$ (we drop the prefix when there is no confusion). The signature (the set of all names) of L_i are i-terms.

The semantics of DFOL includes a set of local models and domain relations. For each L_i, there is an interpretation domain Δ_i. Let M_i be the set of all first order models of L_i on Δ_i. We call each $m \in M_i$ a *local model* of L_i. A *domain relation* r_{ij}, where $i \neq j$, is a subset of $\Delta_i \times \Delta_j$. The domain relation r_{ij} represents the capability of the module j to map the objects of Δ_i in Δ_j, or, the j's *subjective view* of the relation between Δ_i and Δ_j. In general, $r_{ij} \neq r_{ji}^{-}$.

We use $\langle d, d'\rangle$ in r_{ij} to denote that from the point of view of j, the object d in Δ_i is *mapped to* to the object d' in Δ_j. $r_{ij}(d)$ denotes the set $\{d' \in \Delta_j | \langle d, d'\rangle \in r_{ij}\}$. For a subset $D \subseteq \Delta_i$, $r_{ij}(D)$ denotes $\cup_{d\in D} r_{ij}(d)$.

Example 1 *An ontology contains two modules $L_{\{1,2\}}$. L_1 contains knowledge about regions and their relations, such as $\forall x, Country(x) \rightarrow Region(x)$ (a 1-formula). L_2 contains knowledge about people, such as $\exists x, European(x) \rightarrow Caucasian(x)$. The local domain Δ_1 has objects* India *and* USA*, and local domain Δ_2 has objects* Hindu*,* Indian*,* Iowan *and* American*. The domain relation r_{12} is $\langle 1 : \mathsf{India}, 2 : \mathsf{Indian}\rangle, \langle 1 : \mathsf{USA}, 2 : \mathsf{American}\rangle, \langle 1 : \mathsf{USA}, 2 : \mathsf{Iowan}\rangle$, while the domain relation r_{21} is $\langle 2 : \mathsf{Hindu}, 1 : \mathsf{India}\rangle, \langle 2 : \mathsf{Indian}, 1 : \mathsf{USA}\rangle, \langle 2 : \mathsf{American}, 1 : \mathsf{USA}\rangle$. $r_{12}(1 : \mathsf{USA}) = \{2 : \mathsf{American}, 2 : \mathsf{Iowan}\}$. Note that L_1 and L_2 hold different semantic points of view, e.g. on the meaning of* Indian*, so that $r_{12} \neq r_{21}^{-}$.*

4 Semantics of Linking – DDL

The linking approach to the integration of ontology modules is aimed at preserving the autonomy of loosely coupled modules, while allowing restricted "mappings" between formulae of linked modules. Formally, a linking approach assumes that a) For any pair of modules L_i and $L_j, i \neq j$, i-terms and j-terms are *disjoint*; b) The semantic connection between L_i and L_j is enabled only by mappings that are interpreted as domain relations $r_{ij} \subseteq \Delta_i \times \Delta_j$; c) Relations within local interpretation domains and inter-module relations are disjoint.

Distributed Description Logics (DDL) [3] is one of the first linking-based modular ontology formalisms. In DDL, the semantic mappings between disjoint modules L_i and L_j are established by a set of "Bridge Rules"(B_{ij}) of the form:

- INTO rule: $i : \phi \xrightarrow{\sqsubseteq} j : \psi$, semantics: $r_{ij}(\phi^{m_i}) \subseteq \psi^{m_j}$
- ONTO rule: $i : \phi \xrightarrow{\sqsupseteq} j : \psi$, semantics: $r_{ij}(\phi^{m_i}) \supseteq \psi^{m_j}$

where $m_i(m_j)$ is a model of $L_i(L_j)$, ϕ, ψ are formulae, r_{ij} is a domain relation which serves as the interpretation of B_{ij}. Note that B_{ij} is directional. We will only consider bridge rules between concepts in our discussions since it is the only case that has well-understood semantics and reasoning mechanisms [9].

Distributed concept correspondence between two modules in DDL covers some of the most important scenarios that require mapping between ontology modules. However, the expressivity of DDL is limited in some settings that arise in practical applications: For example, DDL cannot be used to express "a person x works in a region y". Additional semantic difficulties with DDL are noted in [6; 5]: (a) *Subsumption Propagation problem*: concept subsumption links in DDLs do not propagate transitively. For example, in the case of 3 ontology modules $L_{\{1,2,3\}}$, the bridge rules $1 : Bird \xrightarrow{\sqsupseteq} 2 : Fowl$ and $2 : Fowl \xrightarrow{\sqsupseteq} 3 : Chicken$ do not in general ensure that $1 : Bird \xrightarrow{\sqsupseteq} 3 : Chicken$; (b) *Inter-module Unsatisfiability problem*: DDLs may not detect unsatisfiability across ontology modules. For example, $1 : Bird \xrightarrow{\sqsupseteq} 2 : Penguin$ and $1 : \neg Fly \xrightarrow{\sqsupseteq} 2 : Penguin$ do not render $2 : Penguin$ unsatisfiable even if L_1 entails $Bird \sqsubseteq Fly$.

Such difficulties are rooted in a fundamental assumption of DDL: local modules are disjoint. Therefore, a bridge rule cannot be read as concept subsumption, such as $i : A \sqsubseteq j : B$. Instead, it must be read as: ([3])

- $i : A \xrightarrow{\sqsubseteq} j : B \Rightarrow (i : A) \sqsubseteq \forall R_{ij}.(j : B)$
- $i : A \xrightarrow{\sqsupseteq} j : B \Rightarrow (j : B) \sqsubseteq \exists R_{ij}^{-}.(i : A)$

where R_{ij} is a new role representing correspondences B_{ij} between L_i and L_j. Therefore, for the given subsumption propagation example, if $B_{13} = \emptyset$, entailment $Chicken \sqsubseteq \exists R_{13}^{-}.Bird$ is not always true. For the inter-module unsatisfiability problem, concept $Penguin$ ($\sqsubseteq \exists R_{12}^{-}.(Fly) \sqcap \exists R_{12}^{-}.(\neg Fly)$) is satisfiable.

In the absence of a principled approach to avoiding arbitrary domain relations, all semantic relations (bridge rules) between DDL modules are *localized* to pairs of modules that are bridged by the rules in question. Consequently, they cannot be safely reused by other modules, thereby precluding subsumption propagation. Note further that in order to enable distributed (not necessarily exact) reasoning in general, a DDL KB needs explicit declaration of domain relations between *each* pair of modules, leading to an exponential blowup in the number of bridge rules, with the attendant inefficiency and increased risk of inconsistencies. In general, DDL, as presented in [3], meets the *localized semantics* and *directional semantic relations* requirements, but *not* the *exact reasoning* and *transitive reusability* requirements.

Serafini et al. [9] asserted that the inter-module unsatisfiability difficulty is the result of incomplete modelling. They argued that it can be eliminated if extra information, for example, $1 : \neg Bird \xrightarrow{\sqsubseteq} 2 : \neg Penguin$ and $1 : Fly \xrightarrow{\sqsubseteq} \neg 2 : Penguin$, is added to guarantee one-to-one domain relations. Our investigation reveals a more general result: In order to avoid both the difficulties of DDL noted in [6; 5], namely, subsumption propagation problem and inter-module unsatisfiability problem and to ensure exact DDL inference, domain relations among local models should be one-to-one (i.e. a necessary condition):

Theorem 1 *For a DDL KB $\Sigma_d = \langle \{L_i\}, \{B_{ij}\} \rangle$, if all reasoning problems in Σ_d are to be exact, all interpretations r_{ij} of B_{ij} must be one-to-one.*

Proof Sketch: If reasoning in Σ_d is exact, all modules should agree with what is "nothing" (while can still disagree on what is "everything"). Formally, for any model $m_d = \langle \{m_i\}, \{r_{ij}\} \rangle$ of Σ_d, for any module $L_i, L_j (i \neq j)$, $r_{ij}(\bot_i)$ should be empty, where $\bot_i$ is the short for $C \sqcap \neg C$ for any concept C in L_i. Therefore, for any $x, y \in \Delta_i, x \neq y$, $r_{ij}(x) \cap r_{ij}(y)$ must be empty. That means any object in Δ_j has at most one pre-image in Δ_i. Similarly, $r_{ij}^{-}(\bot_j)$ should also be empty, therefore any object in Δ_i has at most one image in Δ_j. Consequently, any r_{ij} must be one-to-one.

At present, there is no principled approach to coming up with such domain relations. Adding $\neg C \xrightarrow{\sqsubseteq} \neg D$ for each $C \xrightarrow{\sqsupseteq} D$, as suggested in [9], does not necessarily result in injective domain relations for any inter-module concept relations. In general, DDL in its present form does not provide a satisfactory formalism for inter-module, or inter-ontology, subsumption.

5 Semantics of Linking – $\mathcal{E}$-connections

While DDL allows only one type of domain relations, the $\mathcal{E}$-connection approach allows multiple "link" relations between two domains, such as *worksIn* and *bornIn* between $2 : Person$ and $1 : Region$. $\mathcal{E}$-connections between ADSs [8], and in particular, between DLs [7; 6], restrict the local domains of the $\mathcal{E}$-connected ontology modules to be disjoint. Roles are divided into disjoint sets of *local roles* (connecting concepts in one module) and *links* (connecting inter-module concepts). Formally, given ontology modules $\{L_i\}$, an (one-way binary) link $E \in \mathcal{E}_{ij}$, where $\mathcal{E}_{ij}, i \neq j$ is the set of all links from the module i to the module j, can be used to construct a concept in module i, with the syntax and semantics specified as follows:

- $\langle E \rangle (j : C)$ or $\exists E.(j : C)$: $\{x \in \Delta_i | \exists y \in \Delta_j, (x, y) \in E^M, y \in C^M\}$
- $\forall E.(j : C)$: $\{x \in \Delta_i | \forall y \in \Delta_j, (x, y) \in E^M \rightarrow y \in C^M\}\}$

where $M = \langle \{m_i\}, \{E^M\}_{E \in \mathcal{E}_{ij}} \rangle$ is a model of the $\mathcal{E}$-connected ontology, m_i is the local model of L_i; C is a concept in L_j, with interpretation $C^M = C^{m_j}$; $E^M \subseteq \Delta_i \times \Delta_j$ is the interpretation of a $\mathcal{E}$-connection E. $\mathcal{E}$-connections also permit number restrictions on links [8].

A $\mathcal{E}$-connection model M can be mapped to a DFOL model $M_d = \langle\{m_i\}, \{r_{ij}\}\rangle$ with each E^M ($E \in \mathcal{E}_{ij}$) acts as a domain relation r_{ij} [10]. Extending the semantics of $\mathcal{E}$-connection axioms ((1) and (3) below) given in [10] so as to allow the use of constructed concepts ($\exists E.D$ and $\forall E.D$) on either side of the subsumption, we have:

1. $C \sqsubseteq \forall E.D : E^M(C^{m_i}) \subseteq D^{m_j}$
2. $C \sqsupseteq \forall E.D : \forall x \in \Delta_i, E^M(x) \subseteq D^{m_j} \rightarrow x \in C^{m_i}$
3. $C \sqsubseteq \exists E.D : C^{m_i} \subseteq (E^M)^-(D^{m_j})$
4. $C \sqsupseteq \exists E.D : C^{m_i} \supseteq (E^M)^-(D^{m_j})$

where $(E^M)^-$ is the inverse of E^M, C is an i-concept and D is a j-concept.

It has been argued that $\mathcal{E}$-connections are more expressive than DDL [8; 5] because DDL can be reduced to $\mathcal{E}$-connections. However, the reduction as presented in [8; 5] ($C \xrightarrow{\sqsubseteq} D$ to $\langle E\rangle.C \sqsubseteq D$ and $C \xrightarrow{\sqsupseteq} D$ to $\langle E\rangle.C \sqsupseteq D$) , is not semantically sound in light of the DDL and EC semantics in the DFOL framework. We show that *inverse links* being allowed is a necessary condition for $\mathcal{E}$-connections to be more expressive than DDL bridge rules:

Theorem 2 *$\mathcal{E}$-connections, as presented in [8; 5] is strictly more expressive than DDL as presented in [3], only if inverse links are allowed.*

Proof Sketch: Comparison of the semantics of DDL bridge rules and $\mathcal{E}$-connections, if we treat the only domain relation in DDL as a $\mathcal{E}$-connection E, as shown in [10; 3], $C \sqsubseteq \forall E.D$ has the same semantics as the "into" rule $C \xrightarrow{\sqsubseteq} D$ ($r_{ij}(C^{m_i}) \subseteq D^{m_j}$). However, onto rules, such as $C \xrightarrow{\sqsupseteq} D$ ($r_{ij}(C^{m_i}) \supseteq D^{m_j}$), can be translated into $D \sqsubseteq \exists E^-.C$ only if the inversion of $\mathcal{E}$-connections is allowed.

Thus, the language $\mathcal{C}^{\mathcal{E}}_{\mathcal{HI}}(\mathcal{SHIQ}, \mathcal{SHOQ}, \mathcal{SHIO})$ is more expressive than DDL but $\mathcal{C}^{\mathcal{E}}_{\mathcal{HQ}}(\mathcal{SHIQ}, \mathcal{SHOQ}, \mathcal{SHIO})$ [5] is *not*.

$\mathcal{E}$-connections allow multiple relations, construction of new concepts, and transitive links[5]. However, the applicability of $\mathcal{E}$-connections in practice is limited by the need to ensure that the local domains are disjoint: a concept cannot be declared as subclass of another concept in a foreign module thereby ruling out the possibility of asserting inter-module subsumption. Furthermore, a property cannot be declared as sub-relation of a foreign property, and neither foreign classes nor foreign properties can be instantiated. This also presents difficulties in using the OWL importing mechanism [5]. Furthermore, to the best of our knowledge, in general, the exactness of reasoning in $\mathcal{E}$-connected ontologies w.r.t. their centralized counterpart is still unknown.

6 Semantics of Importing – P-DL

Our investigation of the semantics of DDL and $\mathcal{E}$-connections suggest that many of the semantic difficulties of linking approaches might be the result of a fundamental assumption that the local language and local models are disjoint. Thus, it is interesting to consider formalisms for integrating ontology modules that relax this assumption.

OWL (http://www.w3.org/TR/owl-semantics/) does not make such module disjointness assumption and adopts an *importing* mechanism to support integration of ontology modules. However, the importing mechanism in OWL, in its current form, suffers from several serious drawbacks: (a) It directly introduces both terms and axioms of the imported ontologies into the importing ontology, and thus fails to support *local semantics* (b) It provides no support for *partial reuse* of an ontology module.

Package-based Description Logics (P-DL)[2] offer a tradeoff between the strong module disjointness assumption of DDL and $\mathcal{E}$-connections, and on the other hand, the OWL importing mechanics, which forces *complete overlapping* of modules. In P-DL, an ontology is composed of a collection of modules called *packages*. Each term (name of a concept, a property or an individual) and each axiom is associated with a *home package*. A package can use terms defined in other packages i.e., *foreign terms*. If a package L_j uses a term $i:t$ with home package L_i $(i \neq j)$, then we say t is *imported* into L_j, and the importing relation is denoted as r_{ij}^t. In what follows, we will examine a restricted type of package extension which only allows import of concept names. We will show that this restricted form of package extension is not trivial and is more expressive than DDL and $\mathcal{E}$-connection.

The semantics of P-DL is expressed in DFOL as follows: For a package-based ontology $\langle \{L_i\}, \{r_{ij}^t\}_{i \neq j} \rangle$, a distributed model is $M = \langle \{m_i\}, \{(r_{ij}^t)^M\}_{i \neq j} \rangle$, where m_i is the local model of module i, $(r_{ij}^t)^M \subseteq \Delta_i \times \Delta_j$ is the interpretation for the importing relation r_{ij}^t, which meets the following requirements:

- Every importing relation is one-to-one and complete in that it maps each object of t^{m_i} to a single unique object in t^{m_j}, therefore $(r_{ij}^t)^M(t^{m_i}) = t^{m_j}$.
- Each object in the model of a source package corresponds uniquely to an object in the model of any target package for any interpretation of importing relations, i.e., for any $i : t_1 \neq i : t_2$ and any $x, x_1, x_2 \in \Delta_i$, $(r_{ij}^{t_1})^M(x) = (r_{ij}^{t_2})^M(x)$ and $(r_{ij}^{t_1})^M(x_1) = (r_{ij}^{t_2})^M(x_2) \neq \emptyset \rightarrow x_1 = x_2$.
- Compositional Consistency: if $(r_{ik}^{i:t_1})^M(x) = y_1$, $(r_{ij}^{i:t_2})^M(x) = y_2$, $(r_{jk}^{j:t_3})^M(y_2) = y_3$, , (where t_1 and t_2 may or may not be same), and y_1, y_2, y_3 are not null, then $y_1 = y_3$. Compositional consistency helps ensure that the transitive reusability property holds for P-DL.

The domain relation between m_i and m_j is $r_{ij} = \cup_t (r_{ij}^t)^M$

Lemma 1 *Domain relations in a P-DL model are one-to-one*

Lemma 1 states that a domain relation r_{ij} in a P-DL model isomorphically maps, or copies, the relevant partial domain from m_i to m_j. For any concept $i : C$, $r_{ij}(C^{m_i})$, if not empty, contains the copy of all objects in C^{m_i}. Therefore, if $i : C$ is imported into j, we define inter-module subsumption $i : C \sqsubseteq_P j : D$ as $r_{ij}(C^{m_i}) \subseteq D^{m_j}$ and $i : C \sqsupseteq_P j : D$ as $r_{ij}(C^{m_i}) \supseteq D^{m_j}$. Note that inter-module subsumption is substantially different from bridge rules in DDL. DDL bridge rules are bridging semantic gaps between *different* concepts, and there is no principled way to ensure subjective domain relations to be semantically consistent (such as one-to-one mappings). In contrast, P-DL importing mechanism bridges the semantic gaps between multiple references of the *same* concept in different modules. Importing of C from i to j cannot be reduced to a DDL equivalency bridge rule $C \xrightarrow{\equiv} C'$, since in DDL $r_{ij}(C^{m_i}) = C'^{m_j}$ does not guarantee C^{m_i} and C'^{m_j} are interpretations for the same concept.

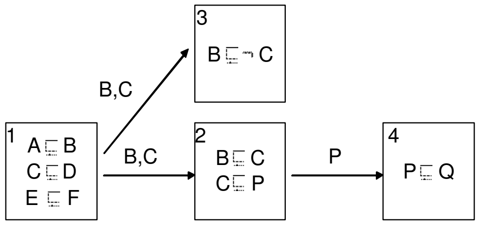

Figure 1: P-DL Ontology Example

P-DL domain relations allow us to relax the domain disjointness assumption adopted in DDL and $\mathcal{E}$-connections, since the construction of a local model is dependent on the structure of local models of imported modules. However, the loss of disjointness does not sacrifice *localized semantics* property of modules, since they are (unlike in the case of OWL import mechanism which requires the local models to be completely overlapping), only partially overlapping. The semantics of the part of a module that is not exported to any other module remains local to that module. Consequently, there is no required global model. The example below demonstrates that P-DL also satisfies directional semantic relation and module transitive reusability properties.

Example 2 *Consider four modules $L_{\{1,2,3,4\}}$ as shown in Figure 1.*

1. *Transitivity of inter-module subsumption holds: $r_{14}(A^{m_1}) = r_{24}(r_{12}(A^{m_1})) \subseteq r_{24}(r_{12}(B^{m_1})) = r_{24}(B^{m_2}) \subseteq r_{24}(C^{m_2}) \subseteq r_{24}(P^{m_2}) = P^{m_3} \subseteq Q^{m_3}$, i.e., $A \sqsubseteq_P Q$. Although no term in L_1 is directly imported into L_4, we can infer the domain relation r_{14} from $r_{12j} \circ r_{24}$ utilizing their compositional consistency property.*
2. *The importing relation is directional. Thus, $r_{12}(A^{m_1}) \subseteq r_{12}(D^{m_1})$ is enforced only in L_2, while $A^{m_1} \subseteq D^{m_1}$ is not required in L_1. There is no information "backflow" in importing. Therefore, while L_2 and L_3 are inconsistent, they are all consistent to L_1, and the consistency of L_1 is still guaranteed.*
3. *The model overlapping is only partial. For instance, E and F in i are semantically separated from L_2 and have no correspondence in the local model m_2.*

Because of the isomorphic and complete nature of importing relations, we have:

Theorem 3 *Reasoning in a P-DL KB is exact w.r.t. its centralized counterpart.*

Proof Sketch: A reduction from a P-DL KB to a classic KB can be easily accomplished by merging imported terms. Since the only type of semantic relations between modules are importing relations, and shared terms are always interpreted consistently in different modules, we can transform a distributed P-DL model into a classic DL model by merging all "copied" objects in each of the local models. Therefore, any satisfiability or entailment problem in P-DL is exact relative to its centralized counterpart.

P-DL despite its stronger domain relation restrictions, is more expressive than DDLs and $\mathcal{E}$-Connections. For example, an into rule $i : C \xrightarrow{\sqsubseteq} j : D$ in DDL can be reduced to a P-DL axiom $C \sqsubseteq D$ in module j and C is an imported concept; A $\mathcal{E}$-connection-like constructed concept such as $\exists(i : E).(j : D)$ can be defined in the module i, where $j : D$ is imported into i, with semantics: $\{x \in \Delta_i | \exists y \in \Delta_j, y' = r_{ji}(y) \in \Delta_i, (x, y') \in E^{m_i}, y \in D^{m_j}\}\}$. $\forall(i : E).(j : D)$ can be constructed similarly.

However, a limitation of the importing approach adopted by P-DL is that the general decidability transfer property does not always hold in P-DL since the union of two decidable fragments of DL may be undecidable [1]. This presents semantic difficulties in the general setting of connecting ADSs [1; 8]. However, in a setting where different ontology modules are specified using subsets of the *same* decidable DL language, such as $\mathcal{SHOIQ}(D)$ (OWL-DL), the union of such modules is decidable. Hence, we believe that P-DL offers an attractive alternative compromise between the linking approaches such as $\mathcal{E}$-connections and DDL on the one hand and all-or-none importing mechanism of OWL.

7 Conclusions

In this paper, we investigate the semantics of DDL, $\mathcal{E}$-connections and P-DL, and show that (a) one-to-one domain relation is a necessary condition for exact DDL reasoning; (b) $\mathcal{E}$-connections, in general, are *not* more expressive than DDL 3) show how an importing approach in P-DL can be used to ensure transitivity of inter-module subsumption without sacrificing the exactness of inference in P-DL with only a minor compromise of local semantics. Our results raise the possibility of avoiding many of the semantic difficulties in current modular ontology language proposals by removing the strong assumption of module disjointness.

References

[1] F. Baader, C. Lutz, H. Sturm, and F. Wolter. Fusions of description logics. In *Description Logics*, pages 21–30, 2000.

[2] J. Bao, D. Caragea, and V. Honavar. Towards collaborative environments for ontology construction and sharing. In *International Symposium on Collaborative Technologies and Systems (CTS 2006)*. 2006.

[3] A. Borgida and L. Serafini. Distributed description logics: Directed domain correspondences in federated information sources. In *CoopIS/DOA/ODBASE*, pages 36–53, 2002.

[4] C. Ghidini and L. Serafini. *Frontiers Of Combining Systems 2, Studies in Logic and Computation*, chapter Distributed First Order Logics, pages 121–140. Research Studies Press, 1998.

[5] B. C. Grau. *Combination and Integration of Ontologies on the Semantic Web.* PhD thesis, Dpto. de Informatica, Universitat de Valencia, Spain, 2005.

[6] B. C. Grau, B. Parsia, and E. Sirin. Working with multiple ontologies on the semantic web. In *International Semantic Web Conference*, pages 620–634, 2004.

[7] O. Kutz, C. Lutz, F. Wolter, and M. Zakharyaschev. E-connections of description logics. In *Description Logics Workshop, CEUR-WS Vol 81*, 2003.

[8] O. Kutz, C. Lutz, F. Wolter, and M. Zakharyaschev. E-connections of abstract description systems. *Artif. Intell.*, 156(1):1–73, 2004.

[9] L. Serafini, A. Borgida, and A. Tamilin. Aspects of distributed and modular ontology reasoning. In *IJCAI*, pages 570–575, 2005.

[10] L. Serafini, H. Stuckenschmidt, and H. Wache. A formal investigation of mapping language for terminological knowledge. In *IJCAI*, pages 576–581, 2005.