Predictive Model's Development Based on the Wavelet Analysis Technique

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Abstract. In the paper, associative search identification models in variable structure control systems are considered. The methods of non-linear systems identification by using the multiple-scale wavelet transform are presented. A methodology of a variable structure control system's synthesis is presented based on information models. Issues of the stability of a model built by use of the associative search are considered, in the aspect of the spectrum analysis of the multi-scale wavelet expansion. Methods based on the wavelet analysis are characterized with a unique possibility to select "frequency-domain windows".

Keywords: system identification, predictive model, information model, virtual model, associative search, fuzzy model, knowledge base, wavelets.

1 Preliminaries

Variable structure control systems (VSCS) are a class of control systems, providing an effective possibility to solve main problems of the control theory – stabilization problem and tracking problem – by use of automatically switched algorithms.

VSCS provide a rational system performance in accordance to a set dynamics without constructing an adaptive system model (structure and parametric one). In paper [1], the control invariance with respect to parametric disturbances was proven.

The methodology of the synthesis of variable structure systems is based on introducing a fundamental notion of new kind feed-backs (operator and coordinate-operator ones). The approach is forming an operator (an algorithm of forming a control implemented by a controller) transforming signals-coordinates (functions in the time) as an element of a set of stabilizing feed-backs. A parameterization of the set enables one to impose a one-to-one relationship between the signals-coordinates and signals-operators.

In paper [1], a constructive approach to the design of such algorithms is presented, enabling the sliding mode, for second order linear systems. However, the VSCS ideolo-

In: A. Kononov et al. (eds.): DOOR 2016, Vladivostok, Russia, published at http://ceur-ws.org

gy provides a possibility to design control systems of such a kind under the conditions of the parametric uncertainty not for linear dynamic plants only. In the present paper, a possibility of applying the principle of the operator feed-backs for non-linear as well as time-varying systems is demonstrated.

To solve such a problem, one assumes under forming the control algorithm at each time instant (what corresponds to the definition of a variable structure system) applying information on the system status/state (current and archive one), in other words: application of all its dynamic previous history. Meanwhile, not a conventional control system with a feed-back identifier is formed: a system with operator feed-back is formed, in which the signal-operator is formed by use of a virtual plant model created at each time instant on the basis of intelligent analysis of persistently updated date on the plant dynamics.

2 Variable structure systems using inductive knowledge

Let us consider a scheme of a control system with additional dynamic error feed-back, displayed in Fig. 1.

In systems investigated in [1], the operator $\mathbf{S}(t)$ produced the sign change. As to the general case, in this block some, generically, non-linear transformation of the signal s is implemented:

$$r(t) = \mathbf{S}\big(s(t), \sigma(t)\big). \tag{1}$$

Here **S** is a non-linear operator that is formed on the basis of analysis a specific control problem. In the most general case, **S** may be, for instance, logic production operator implementing both conventional and fuzzy control.

To form at each time instant t the signal-operator r producing varying the structure of the regulator **R**, besides the dynamic error $\sigma(t)$ one should use additional a priori information on the system. In this connection, let us recollect that the unique information code sets the system state, its information "portrait". How can one use this information (interpreted as the signal $s(t)$), and how is $s(t)$ formed?

$s(t)$ is to provide to **S** an intelligent (based on knowledge, that is regularities, formed and updated at each time instant on the basis of the history data analysis) prediction information model of the plant. This model provides to **S** an "information support" to make a decision by the system on changing the controller structure. Based on this support, the signal-operator $r(t)$ is formed.

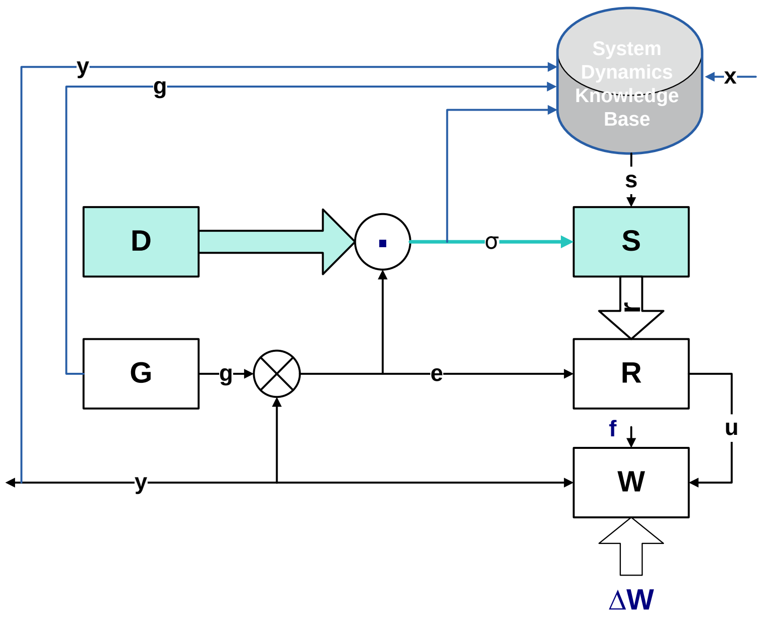

Fig. 1. Scheme of control system with additional dynamic error feed-back

Taking into account the fact that the same value of the dynamic error $\sigma(t)$ may take place under different sets of signal values (involving operator ones), we come to the inference that the signal $s(t)$ may be represented as a regression (in the general case, nonlinear) of the following variables:

$$s(t) = \mathbf{F}[(g(t), g(t-1), \dots, g(t_0),\\ \sigma(t-1), \dots, \sigma(t_0), y(t-1), \dots, y(t_0), (t-1), \dots, u(t_0), t]. \quad (2)$$

A principal distinction of the intelligent model from a conventional predicting model is the following. The intelligent (that is based on knowledge revealed from the data analysis) model is, in its entity, a situational ("virtual") one, that is accounting particularities of the current state, hence, at each time instant this model may have a new structure. This its attribute enables one to synthesize VSCS for non-linear, and, involving, time-varying systems.

The operator **F** (also, in the general case, being non-linear) is to be formed by use an approach based on the data analysis. The history system data analysis will enable one to reveal certain regularities (inductive knowledge). An example of such knowledge may consist in relating the system input (a vector in corresponding vector space) at each current time instant t to a certain domain of this space (clustering).

To determine $s(t)$, in particular, one may apply the technique of the ***associative search*** [2], being a search method based on the analysis of the previous history of state dynamics of a plant under study and constructing *virtual models*.

3 The associative search method of virtual model development

Algorithms based on knowledge revealed from history system data (inductive knowledge, persistently enriched) implement an intelligent approach to constructing identification models. The intelligence is applying knowledge (*Knowledge Based*) revealed from history data on the basis of their analysis (*Data Mining*).

The process of knowledge processing in the intelligent system is reduced to recovering (associative search of) *knowledge* over its fragment. Meanwhile, the *knowledge* may be interpreted as associative connections between *images*. As an image, we will use "sets of indicators", that is components of input vectors, input variables.

The criterion of closeness between images may be formulated in very different manners. In the most general case, it may be represented as a logic function, the predicate. In a particular case, when sets of indicators are vectors in *n*-dimensional space, the criterion of closeness may be a distance in this space.

The associative search process may be implemented either as a process of recovering the image over partially given indicators (or recovering a knowledge fragment under the conditions of incomplete information; as a rule, just this process is simulated in different models of the associative memory), or as a process of searching another images that are associatively connected with the given one, attached to other time instants.

In papers [2-4], an approach to form the support on decision making on the control is proposed, based on dynamic modeling the associative search procedure. Results of adoption of the associative search algorithms developed by the authors for industrial processes of the chemical and petroleum manufacturing, processes of control in intelligent power networks (smart grids), trading processes, transport logistic processes.

The method of the *associative search* consists in constructing *virtual* predicting models. The method assumes constructing predicting model of a dynamic plant, being new under each *t*, by use of a set of history data ("associations") formed at the stage of learning and adaptively corrected in accordance to certain criteria, rather than approximating real process in the time.

Within the present context, linear dynamic model is of the form:

$$y_N = \sum_{i=1}^{m} a_i y_{N-i} + \sum_{j=1}^{r_s} \sum_{s=1}^{S} b_{j,s} x_{N-j,s}, \quad \forall j = \overline{1, N}, \tag{3}$$

where: y_N is the prediction of the output plant at the time instant N, x_N is the vector of input actions, m is the memory depth in the output, r_s is the memory depth in the input, S is the dimension of the input vectors, a_i and $b_{j,s}$ are the tuned coefficient, meanwhile $x_{N-j,s}$ are selected disregarding the order of the chronological decreasing, have been referred as the *associative pulse.*

Let us note that this model is not classical regression one: there are selected certain inputs in accordance to a certain criterion, rather than all chronological "tail".

The algorithm of deriving the virtual model consists in constructing at each time instant an approximating hypersurface of the space of input vectors and single-dimensional outputs. To construct the virtual model, corresponding to some time instant, from the archive there are selected input vectors being in a certain sense close to the current input vector. An example of selecting the vectors is described below. The dimension of this hypersurface is selected heuristically. Again, by use of classical (non-recursive) least squares (LS) method there is determined the output value (modeled signal) in the next time instant.

Meanwhile, each point of the global non-linear surface of the regression is formed in the result of using linear "local" models, in each new time instant.

In contrast to classical regression models, for each fixed time instant from the archive there are selected input vectors being close to the current input vector in the sense of a certain criterion (rather than the chronological sequence as it is done in regression models). Thus, in equation (3) r_s is the number of vectors from the archive (from the time instant 1 to the time instant N), selected in accordance to the associative search criterion. At each time segment $[N-1, N]$ there is selected a certain set of r_s vectors, $1 \leq r_s \leq N$. The criterion of selecting the input vectors from the archive to derive the virtual model in the given time instant over the current plant state may be as follows.

Let us introduce as a distance (a norm in $\mathfrak{R}^S$) between points of the S-dimensional space of inputs the value:

$$d_{N,N-j} = \sum_{s=1}^{S} \left| x_{N,s} - x_{N-j,s} \right|, \qquad \forall j = \overline{1, N}, \tag{4}$$

where $x_{N,s}$ are components of the input vector at the current time instant N.

By virtue of a property of the norm ("the triangle inequality", we have:

$$d_{N,N-j} \leq \sum_{s=1}^{S} \left| x_{N,s} \right| + \sum_{s=1}^{S} \left| x_{N-j,s} \right|, \qquad \forall j = \overline{1, N}, \tag{5}$$

Let for the current input vector x_N:

$$\sum_{s=1}^{S} \left| x_{N,s} \right| = d_N. \tag{6}$$

To derive an approximating hypersurface for the vector x_N let us select from the archive of the input data such vectors x_{N-j}, $j = \overline{1, N}$ that for a set D_N the condition will be hold:

$$d_{N,N-j} \leq d_N + \sum_{s=1}^{S} \left| x_{N-j,s} \right| \leq D_N, \qquad \forall j = \overline{1, N}, \tag{7}$$

where D_N may be selected, for instance, from the condition:

$$D_N \geq 2 d_N^{max} = 2 \max_j \sum_{s=1}^{S} \left| x_{N-j,s} \right|. \tag{8}$$

If in the selected domain there will be not enough quantity of inputs to apply the LS method, that is the corresponding system of linear equations will be unsolvable, then the selected criterion of selecting points in the space of inputs may be weakened due to increasing the threshold D_N.

Under the assumptions that the input actions meet the Gauss-Markov conditions, the estimates obtained via the LS method are unbiased and statistically effective.

4 Solving the system of linear equations for the LS method procedure

However, solving the problem of constructing virtual closed-loop models (as well as the conventional identification synthesis) are characterized by considerable difficulties. Under the closed-loop case, the control process is with depended values, and the question on the existence of identifying strategies is not trivial. Optimal controllers generate linear state feed-backs; this leads to an degenerate problem [5].

For the identification of dynamic plants by use of the associative search technique in the degenerate case, Moore-Penrose method [6, 7] is applicable to obtain pseudo-solutions to the system of linear equations under using the least squares procedure. Applying the method to solve the identification problem is presented in paper [8].

5 Clustering based associative search

In order to increase the computation capability at the stage of learning and under subsequent real plant performance, one of the data mining methods – clustering (dynamic classification, automated grupping data, unsupervised learning) is used. There are known numerous such methods: hierarchical algorithms, k-means algorithm, minimal covering tree algorithm, nearest neighbor method, etc. All they determine (in the dependence on the time, in contrast to the classification) the belonging of the point in the multi-dimensional space by one of the domain, which the space is partitioned on.

As a result, each investigated point in the multi-dimensional space may be related to a group by assigning a cluster label to it. In the problem of the associative search to select input vectors being "close" to the current one, the cluster label is determined in accordance to the associative pulse (the criterion of the associative selection), and to derive the virtual models the vectors are selected inside the corresponding cluster.

In the problem of determining the signal-operator, the method of the associative search enables one to predict $r(t)$ on the basis of data on belonging the point $s(t)$ to one of the clusters of the space

$$\mathbf{S}\{g(t), g(t-1), \dots, g(t_0), \sigma(t), \dots, \sigma(t_0), y(t-1), \dots, y(t_0), \\ u(t-1), \dots, u(t_0), t\}. \tag{9}$$

Such an approach enables one to avoid the need of information on the structure of the control plant $\mathbf{W}$. The plant is even not needed to be linear. The control quality will depend on the amount of analyzed and clustering criterion.

6 Data Mining techniques in associative search tasks

The use of a priori information (process knowledge) plays the key role in the intelligent approach to predictive model building described above. Therefore, the intelligent data analysis, in particular, the selection of a data set meeting a certain criterion ("associative impulse") at each algorithmic step, plays the key role. In the simplest case, the associative impulse presumes the selection of the next vector from the history (this selection operation is called *association*) in such a way that this vector belongs to a certain domain in the space of vectors stored in the process history. A certain metric is introduced respectively.

Associative search algorithm discussed above is effective for the case, when the control plant is nonlinear, high response speed is not required, and the computational resources enable the exhaustive search of the process history. Such an approach is quite satisfactory for the identification of a rather broad class of control plants, for example, continuous and semi batch processes in chemical and oil refining industries.

However, the unpractical use of computational resources within this approach is obvious. Worse yet, each data search cycle requires to pick out the number of vectors sufficient for solving a system of equations, which, according to the LSM, allows to predict the output at the next time step. It is not guaranteed that such selection (even with high redundancy) is attainable in a single step.

7 Application of traditional clusterization methods for the associative search

To increase the algorithm speed (which is a key performance index of such algorithms for certain applications) and computational resource saving, it is proposed to teach the system. Clustering (learning without a teacher) looks an effective technique.

When using any of the known clustering algorithms ("crisp" case), the original set of objects $\bar{x}_i \in \mathrm{X}, i = 1, \ldots, N$ is split into several disjoint subsets. Here, any object from $\mathbf{X}$ belongs to a single class only.

When using fuzzy clustering techniques, it is allowed for one object to belong to several (or all) clusters simultaneously but with different degrees of certainty determined by the selected membership function. Here, the clusters are fuzzy sets. Fuzzy clustering may often be more preferable than crisp, for example, for the objects located at cluster borders.

Associative search problem is solved by clustering technique (both crisp and fuzzy) in the following way.

The current vector under investigation is attributed to a certain cluster per the criterion of minimum distance to the center: $\min_k \sum_{k=1}^{K} \| g_k - \bar{x}_N \|^2$, where $\bar{x}_N \in \mathrm{X}$ is the current input vector of the control plant under investigation.

Further, the vectors are picked out from this cluster, which meet the selected associative search criterion. If the vectors picked out from the cluster are not enough for solving the prediction problem by means of LMS, the cluster can be enlarged by one of the known single-link methods which combine 2 clusters with minimum distance between any 2 of their members.

This naturally ensures a huge saving of computational resources as against the exhaustive search over the whole process history at each step. However, this aggregation into a new cluster presumes that many objects deliberately not meet the associative search criterion.

The approach described below looks the most reasonable.

Virtual clustering ("impostor" method). For each time step N, the current vector under investigation is attributed to a certain cluster per the criterion of the minimum distance to the center – in the same way as with the traditional method.

Assume $\min_k \sum_{k=1}^{K} \|g_k - \bar{x}_N\|^2$ is attained for k=r. Further, $\bar{x}_N$ is assigned to be the center of the cluster $\mathbf{A}_r$. If additional selection is needed from the archive of vectors meeting the associative search criterion, then the clusters are chosen for aggregation with the minimum distance between their centers and the vector $\bar{x}_N$. In such case not only a considerable number of vectors distant from $\bar{x}_N$ will be discarded, but also the maximum possible number of vectors meeting the associative search criterion will appear.

After the associative search is completed, the assignment of $\bar{x}_N$ as the center of the cluster $\mathbf{A}_r$ is cancelled, and the process is continued in the same way as in the traditional algorithm.

8 The methodology of the intelligent VSCS synthesis

The sets of values of signals, forming virtual model (9) in different time instants, and values of $s(t)$, corresponding to them, fill in the Knowledge Base of the system (see Fig. 1). Under accounting the content of the knowledge base, involving values of the signal $\sigma(t)$ one may obtain (by use of the LS method) statistically effective unbiased prediction of the system output. However, under the closed-loop description, it is reasonable the operator **D**, whose main function is to generate the system dynamics, to implement the multiplication of $e(t)$ and values of a limited signal $d(t)$, where $e(t)$ and $d(t)$ are stochastically independent.

The virtual model that is synthesized by virtue of changing the internal system dynamics is the generalized information model of the control plant.

In the event, when data on the coordinate signals are taken into account only, that is in the model coefficients at e, σ are set to be equal to zero, we come to the conventional closed-loop control scheme with an identifier accompanied with all its problems.

In the event, when, wise versa, in the model the coefficients at e, σ are not equal to zero, while the coefficients at g, y, u are equal to zero, we obtain the information model of the internal system dynamics. Such a model will provide vanishing the dynamic error e, σ. However, in the general case, meeting certain control properties is not guaranteed, and, first of all, the stability. In other words, the information plant model permits not only to provide vanishing the dynamic error, but also to keep certain quality of the system performance.

In papers [8, 9], sufficient criteria of the stability of time-varying dynamic plants in the terms of the spectrum of the ***multi-scale wavelet expansion*** of inputs and outputs of the system. And since under deriving virtual predicting models the associative search procedure is used, one may say about the method of the synthesis of VSCS for a broad class of non-linear systems.

More flexible control (in particular, providing the stability), achieved by use of the system dynamics generator, may be obtained due to using the generalized information model under forming $s(t)$ and **S**.

Forming the information model of kind (3), in particular, we obtain the associative virtual model of the dynamics generator.

9 Wavelet approach application to the analysis of non-stationary processes

Within the last two decades, applying the wavelet-transform to the analysis of non-stationary processes has been widely used. The wavelet-transform of signals is a generalization of the spectral analysis, for instance, with regard to the Fourier transform.

First papers on the wavelet analysis of time (spatial) series with a pronounced heterogeneity have appeared in the middle of 1980s [10]. The method was positioned as an alternative to the Fourier transform, localizing the frequencies but not providing the time extension of a process under study. In sequel, the theory of wavelets has appeared and is developed, as well as its numerous applications.

Today the wavelet analysis is used for processing and synthesis of non-stationary signals, solving problems of compressing and coding information, image processing, in the theory and practice of the pattern recognition, in particular, in the medicine, and in many other branches. The practice of applying wavelets was found effective for studying geophysical fields, time meteorological series, forecasting earth quakes. The approach is effective for research of functions and signals being non-stationary in the time are heterogeneous in the space, when results of the analysis are to contain not only the frequency signal characteristics (distribution of the signal power over frequency components), but also information on local coordinates, at which certain groups of the frequency components are manifested, or at which fast changes of the frequency components of the signal are the case.

The wavelet analysis is based on applying a special linear transform of processes to study real data interpreted by these processes, characterizing processes and physical properties of real plants, in particular, technological processes. Such a linear transform is implemented by use of special soliton-like functions (wavelets) forming an orthonormal basis in L^2.

The wavelet transform (WT) of a one-dimensional signal is its representation in the form of the generalized Fourier series or Fourier integral over a system of basis functions. The method is based on the fundamental conception of representation of arbitrary functions on the basis of shifts and extensions of a one localized small wave, or the wavelet function, that quickly decays in the direction to zero. The wavelet is formed in such a manner that the function forming it (the wavelet forming function, or the mother wavelet) is characterized by a certain scale (frequency) and localization in the time due to the operations of the time shift and changing the time scale. The time scale is analogous to the oscillation period, i.e. it is inverse one with regard to the frequency, and the shift interprets the displacement of the signal over the tine axis.

The wavelet transform maps a one-dimensional process into a two-dimensional surface in the three-dimensional space (the frequency and time are considered as independent variables). Thus, properties of the process are studied both in the time and frequency domains, providing a possibility to investigate the dynamics of the frequency process make-up and its local particularities. This enables one to reveal coordinates at which certain frequencies are manifested most considerably.

A graphical interpretation of the wavelet transform may be demonstrated by projecting onto the plane and emphasizing iso-lines characterizing changing intensities of coefficients of the wavelet transform under different time scales, as well as to obtain disposition of local extrema of surfaces.

In comparison to the Fourier transform apparatus, when a function is used generating the orthonormal basis of the space by use of the scale transform, the wavelet transform is formed by use of a basis function localized in a bounded domain and belonging to the space, i.e. to the all numerical axis. In comparison to the «window» Fourier transform, to obtain the transformation on one frequency all time information is not already required. The Fourier transform does not provide information on local properties of a signal under fast enough changes in the time of its spectral make-up. Thus, the wavelet transform may provide one with frequency-time information on a function, which in many practical situations is more actual than information obtained by the standard Fourier analysis.

Wavelets also provide a powerful tool of approximation, which may be used for synthesis of functions that are poorly approximated by other methods, with a minimal number of basis functions.

The wavelet transform, as a mathematical tool, serves, basically, to analyze data in the time and frequency domains. The wavelet theory may be used for the system identification in different aspects. An investigation of the interaction of the identification theory and wavelet analysis was, to some extent, presented in [11]. It was pointed out that wavelets are used, mainly, for the identification of non- linear systems with a particular structure, where unknown time-varying coefficients may be represented as a linear combination of basis wavelet-functions [12, 13]. Besides the direct wavelet analysis, for system identification there may be used bi-orthogonal wavelets [14], wavelet-frames [15], or even wavelet- networks [16].

There exist many different ways of applying wavelets for linear system identification. Preisig [17] studied the identification of systems with a specific input/output structure, in which the parameters are identified via spline-wavelets and their derivatives. In the paper of [18], an extended by use of an orthonormal transformation least squares method is presented in order to revealing useful information from data.

In the present paper the analysis of possibilities of joint applying the wavelet theory is presented.

10 Conditions of the associative model stability in the aspect of the analysis of the spectrum of multi-scale wavelet expansion

Let a predicting associative model of a non-linear time-varying plant meet equation (3). For the selected detail level L for the current input vector $x(t)$, we obtain the multi-scale expansion [9]:

$$x(t) = \sum_{k=1}^{N} c_{L,k}^{x}(t)\varphi_{L,k}(t) + \sum_{l=1}^{L}\sum_{k=1}^{N} d_{l,k}^{x}(t)\psi_{l,k}(t), \tag{10}$$

$$y(t)=\sum_{k=1}^{N} c_{L,k}^{y}(t)\varphi_{L,k}(t)+\sum_{l=1}^{L}\sum_{k=1}^{N} d_{l,k}^{y}(t)\psi_{l,k}(t),$$

where: L is the depth of the multi-scale expansion ($1 \le L \le L_{max}$, where $L_{max} = \lfloor \log_2 N^* \rfloor$ and N^* is the power of the set of states of the system in the System Dynamics Knowledge Base); $\varphi_{L,k}(t)$ – are scaling functions; $\psi_{l,k}(t)$ are the wavelet functions that are obtained from the mother wavelets by the tension/combustion and shift

$$\psi_{l,k}(t)=2^{l/2}\psi_{\text{mother}}(2^{l}t-k)$$

(as the mother wavelets, in the present case we consider the Haar wavelets); l is the level of data detailing; $c_{L,k}$ are the scaling coefficients, $d_{l,k}$ are the detailing coefficients. The coefficients are calculated by use of the Mallat algorithm [10].

Let us expand equation (3) over wavelets:

$$\begin{aligned}
&\sum_{k=1}^{N} c_{Lk}^{y}(t)\varphi_{Lk}(t)+\sum_{l=1}^{L}\sum_{k=1}^{N} d_{lk}^{y}(t)\psi_{lk}(t)= \\
&=\sum_{k=1}^{N}\left(\sum_{i=1}^{m} a_i c_{Lk}^{y}(t-i)\varphi_{Lk}(t-i)\right)+ \\
&+\sum_{l=1}^{L}\sum_{k=1}^{N}\left(\sum_{i=1}^{m} a_i d_{lk}^{y}(t-i)\psi_{lk}(t-i)\right)+ \\
&+\sum_{k=1}^{N}\left(\sum_{s=1}^{S}\sum_{j=1}^{r_s} b_{sj} c_{Lk}^{s}(t-j)\varphi_{Lk}(t-j)\right) \\
&+\sum_{l=1}^{L}\sum_{k=1}^{N}\left(\sum_{s=1}^{S}\sum_{j=1}^{r_s} b_{sj} d_{lk}^{s}(t-j)\psi_{lk}(t-j)\right)
\end{aligned} \tag{11}$$

In papers [8, 9] it was shown that a sufficient condition of the stability of plant (3) (and, hence, also (11)) is as follows: for $\forall k=\overline{1,N}$ meeting the inequalities is to be provided:

- if $m > R,\ R=\max_{s=\overline{1,S}} r_s$, then the condition for the detailing coefficients:

$$\left|a_m d_{lk}^{y}(t-m)\right| < \left|d_{lk}^{y}(t)\right|,$$

  for the approximating coefficients:

$$\left|a_m c_{Lk}^{y}(t-m)\right| < \left|c_{Lk}^{y}(t)\right|,$$

- if $m < R,\ R=\max_{s=\overline{1,S}} r_s$, then the condition for the detailing coefficients:

$$\left|\sum_{s=1}^{S} b_{sR} d_{lk}^{s}(t-R)\right| < \left|d_{lk}^{y}(t)\right|,$$

  for the approximating coefficients:

$$\left|\sum_{s=1}^{S} b_{sR} c_{Lk}^{s}(t-R)\right| < \left|c_{Lk}^{y}(t)\right|,$$

- if $m = R \ne 1,\ R=\max_{s=\overline{1,S}} r_s$, then the condition of the stability for the detailing coefficients:

$$\left|a_m d_{lk}^{y}(t-m)+\sum_{s=1}^{S} b_{sm} d_{lk}^{s}(t-m)\right|<\left|d_{lk}^{y}(t)\right|,$$

for the approximating coefficients:

$$\left|a_m c_{Lk}^{y}(t-m)+\sum_{s=1}^{S} b_{sm} c_{Lk}^{s}(t-m)\right|<\left|c_{Lk}^{y}(t)\right|,$$

– if $m=R=1,\ R=\max_{s=\overline{1,S}} r_s$, then the condition of the stability for the detailing coefficients: $\left|a_1 d_{lk}^{y}(t-1)+\sum_{s=1}^{S} b_{s1} d_{lk}^{s}(t-1)\right|<\left|d_{lk}^{y}(t)\right|$,

for the approximating coefficients:

$$\left|a_1 c_{Lk}^{y}(t-1)+\sum_{s=1}^{S} b_{s1} c_{Lk}^{s}(t-1)\right|<\left|c_{Lk}^{y}(t)\right|.$$

The investigation of the stability by use of the wavelet analysis may be demonstrated by use of the example when model has the input and output memory depth $m=n=1$. For this model, criteria of the stability for the approximating and detailing coefficients will have the form:

$$\left|\frac{a_1 c_{Lk}^{S}(t-1)+b_1 c_{Lk}^{Ia}(t-1)+b_2 c_{Lk}^{Ua}(t-1)}{c_{Lk}^{S}(t)}\right|< \\ <1,$$

$$\left|\frac{a_1 d_{jk}^{S}(t-1)+b_1 d_{jk}^{Ia}(t-1)+b_2 d_{jk}^{Ua}(t-1)}{d_{jk}^{S}(t)}\right|<1,$$

$$1\le j\le L\le L_{max},\ k=\overline{1,N}.$$

In Fig. 2, 3, meeting the stability criterion is displayed for the approximating and detailing coefficients of the sample for the prediction based on the associative search, where L is the depth of the expansion ($L_{max}=14$), and N is the quantity of vectors selected to derive the approximating model by use of the associative search. The simulation results show that for a number of vectors selected the stability criterion is not met for the prediction. Thus, in these instants a non-stationarity is present that requires additional studying.

Investigating expansion coefficients in accordance to the wavelet stability criterion, in entity, provides not only solving the inverse problem of the spectral analysis for the non-linear operator $\boldsymbol{D}$, but also permits to select such control algorithm that it will keep the system stability. Just in such a manner one should form the operator $\boldsymbol{D}$ under solving the problem of the synthesis of the variable structure systems.

11 Conclusions

For non-linear and time-varying dynamic systems, a methodology of the variable structure systems synthesis is proposed, based on deriving operator feed-backs and virtual models of the plant dynamics. The predicting virtual models are based on the intelligent data analysis of the dynamic dossier of the system status. On the basis of approach proposed, it looks possible to synthesize systems meeting set dynamic properties. It is predicted approaching to the stability bounds on the basis of investigating the dynamics of the coefficients of the multi-scale wavelet analysis.

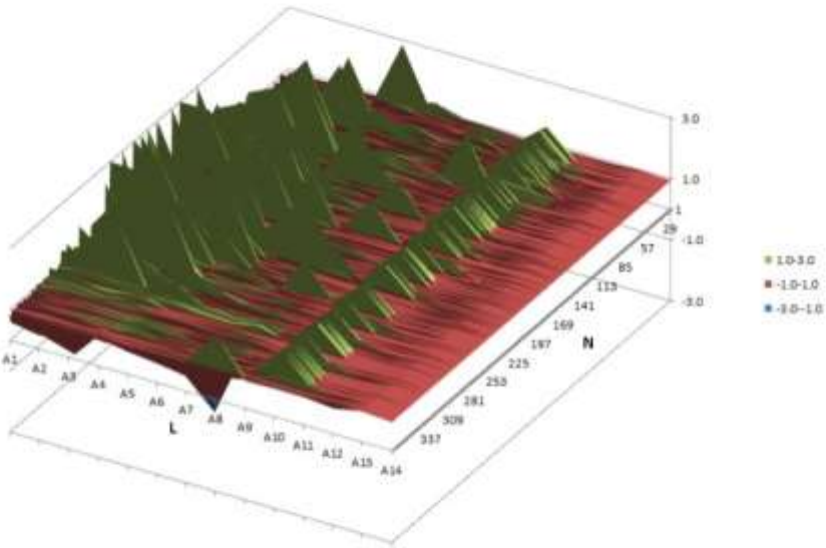

Fig. 2. Stability criterion for the approximating coefficients

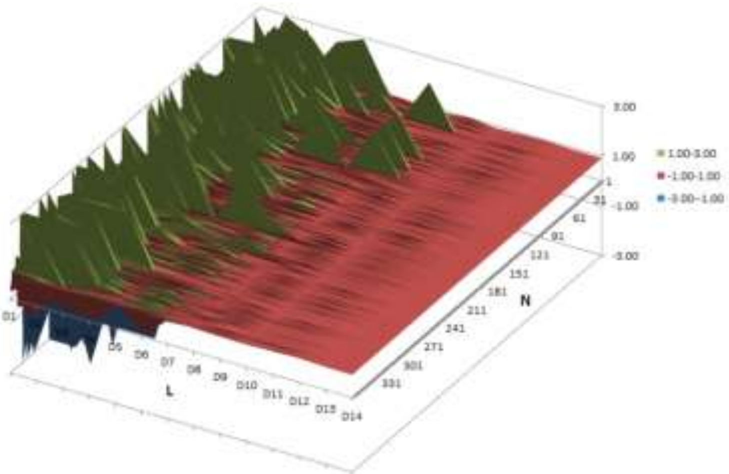

Fig. 3. Stability criterion for the detailing coefficients

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