Concepts as Modal Operators in Description Logics*

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Abstract. Motivated by the colloquial language term "glass gummy bear", an additional type of concept composition for description logics is suggested. This composition type is then axiomatically formalized and called concept generalization. Consistency of the formalization is checked. By proving axiom K and Gödel rule, it is shown that this logic is in fact a multi-modal logic. Concepts could be both modal operators and predicate symbols. A Kripke semantics is presented (the adequacy is future work). In this semantics, the TBox axioms hold for any view, assertions in the ABox hold for the natural view (a selected world in the Kripke structure) only. The relationship to other formalisms is outlined. Further examples are discussed at the end.

1 Motivation

An approach to extend description logics (DL for short) is to find contextual knowledge representations in colloquial language, which have not yet been represented in DL: When colloquial language structures cannot be formalized, they have to be integrated into DL. This paper is about such an integration.

Consider a knowledge base which defines `Glass` and `GummyBear` as concepts. In the TBox, we then have to formalize our knowledge about these terms that gummy bears are sweets that sweets are edible and that glass is not edible:

$$\texttt{GummyBear} \sqsubseteq \texttt{Edible} \tag{1}$$

$$\texttt{Glass} \sqsubseteq \neg\texttt{Edible} \tag{2}$$

My example deals with a glass gummy bear. As we know, a lot of objects can be made out of glass including a piece of glass in the shape of a gummy bear. I know that because I own such an object myself. However, it can by no means be a gummy bear, because a gummy bear is known to be an edible candy. And it is not only a general consensus that gummy bears have to be a candy. This is a defining / essential / necessary characteristic of gummy bears! If we omit this characteristic we will not be able to give a definition of gummy bears to an unknowing person. Nevertheless, a glass gummy bear is by no means a

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contradiction: Such an object is, for example, based on a real gummy bear in form, colour and surface.

So the next task is to represent a special glass gummy bear x in DL: x should be added to the ABox. But how? The introduction of a new concept $\texttt{GlassGummyBear}$ with $\texttt{GlassGummyBear} \sqsubseteq \texttt{Glass}$ in the TBox is not optimal. If we use an intersection $\texttt{Glass} \sqcap \texttt{GummyBear}(x)$ then the knowledge base becomes inconsistent. If we use a union $\texttt{Glass} \sqcup \texttt{GummyBear}(x)$ then we are inaccurate, because $\texttt{Glass} \sqcup \texttt{GummyBear}$ only means that something must be either glass or gummy bear (or both, but this is excluded by the TBox).

There is one formalization approach remaining: We could treat glass not as a concept but a role $\texttt{isGlassModelOfObject}$. So

$$\texttt{isGlassModelOfObject}(x, y) \tag{3}$$
$$\texttt{GummyBear}(y) \tag{4}$$

would be fine in the ABox, where y is the real gummy bear, which is modeled by x. All glass gummy bears are then returned by the following query:

$$\exists \texttt{isGlassModelOfObject}.\, \texttt{GummyBear} \tag{5}$$

However, what if there is no real gummy bear which was modeled by the glass gummy bear. Or if the real gummy bear is unknown?

The general problem is, that the “glass gummy bear” has no connection (role, relation) to a specific gummy bear, only a relation to the concept of the gummy bear. Hence you would need a meta-relation between an individual glass gummy bear and the concept of a glass gummy bear.

My proposal is to extend DL by a further mechanism for the composition of concepts: The following definition describes a glass gummy bear x without modification of the original TBox (2), by

$$\texttt{Glass} \multimap \texttt{GummyBear}(x) \tag{6}$$

in the ABox. This knowledge base is consistent. As we will see later, we can derive $\texttt{Glass}(x)$ and by (2) $\neg\texttt{GummyBear}(x)$. For simplicity, this paper extends only a subset of the DL $\mathcal{ALC}$ without quantifiers and roles. The notation $\multimap$ is adopted from [9].

2 Proof Theoretic Definition

Definition 1 (Concept Generalization). *Let A be a atomic concept and D, E be concepts (meta variables that can be used for any concept expression). Then $A \multimap D$ is a concept (say ”A generalized D”) and is defined by the following* *axioms and the generalization rule:*

$$A \multimap (D \sqcup E) \sqsubseteq A \multimap D \sqcup A \multimap E \qquad \textit{Union} \qquad (7)$$

$$A \multimap \top \equiv A \qquad \textit{Universality} \qquad (8)$$

$$A \multimap \bot \sqsubseteq \bot \qquad \textit{Inconsistent Concept} \qquad (9)$$

$$\top \multimap C \sqsubseteq C \qquad \textit{Realness} \qquad (10)$$

$$A \multimap (\exists r.\ C) \sqsubseteq \exists r.\ (A \multimap C) \qquad \textit{Quantification} \qquad (11)$$

$$\frac{D \sqsubseteq E}{A \multimap D \sqsubseteq A \multimap E} \qquad \textit{Generalization Rule} \qquad (12)$$

Note that the generalization symbol $\multimap$ *has the strongest bond after* $\neg$.

Why these axioms? First of all, it is good that we have an axiom for each possible operand. This makes it easier to define a recursive semantics afterwards. But then, the axioms have to be defended: The union (7) might be most difficult to defend, because we hardly have a union operator in English. Something which is either a leopard or a tiger could be called a tiger-or-leopard. It is reasonable that a stone tiger-or-leopard is a stone tiger or a stone leopard.

The next three axioms deal with the well known predefined concepts $\bot$ and $\top$. Universality (8) simply states that a concept is equivalent when used as an adjective to the universal concept $\top$. There is no reason that the generalization $C \multimap \bot$ of the empty concept $\bot$ would be more than empty, hence (9). But how to read expressions like $\top \multimap C$, where $\top$ is used as an adjective? The only interpretation I have is that these expressions would correspond to "real C", for example a "real gummy bear" is at most a "gummy bear". This concludes (10).

$\mathcal{ALC}$ without roles is not $\mathcal{ALC}$. Considering the veggie burger example in Section 5, Axiom (11) states that roles are not affected by the use of adjectives and that we can transit from the concept "has ingredient meat relative to the veggie property" $\texttt{Veggie} \multimap \exists\texttt{ingredient}.\ \texttt{Meat}$ to "has ingredient veggie meat" $\exists\texttt{ingredient}.\ \texttt{Veggie} \multimap \texttt{Meat}$. Note that axiom (11) is not compatible with the proposal of my original submission, which would also require roles to be modified by adjectives, e.g. $\texttt{Veggie} \multimap \texttt{ingredient}$. But what is the meaning of the `ingredient` relationship relative to the `Veggie` concept? The current version avoids relative roles and makes it easier to provide a semantics by separating roles and concepts.

The generalization rule (12) is the most powerful part of the formalization. If we have a TBox axiom $D \sqsubseteq E$ we automatically have the axiom $A \multimap D \sqsubseteq A \multimap E$ for any (atomic) concept A. Under a different viewpoint the same axioms should hold. For example, a $\texttt{Glass} \multimap \texttt{GummyBear} \sqsubseteq \texttt{Glass} \multimap \texttt{Edible}$ or $\texttt{Veggie} \multimap \texttt{Burger} \sqsubseteq \texttt{Veggie} \multimap \exists\texttt{ingredient}.\ \texttt{Meat}$.

Theorem 1 (Consistency). *There are (infinitly) many knowledge bases (ABox and TBox) in the extended logic where* $\bot \equiv \top$ *is not a consequence, because knowledge base and definition 1 are satisfiable.*

Proof. We can transform the extension back to the original logic by replacing the new $\multimap$ character with $\sqcap$. The transformed axioms (except (11)) can be proven in $\mathcal{ALC}$, moreover the transformed generalization rule is provable. Hence any knowledge base in the extended logic without roles inherits the models of the transfered knowledge base.

$A \sqcap D \sqsubseteq A \multimap D$ does **not** follow from definition 1, but $A \multimap D \sqsubseteq A \sqcup D$ by (13). For example, we have (real) "gummy bear' which imitates a Haribo gummy bear, hence it is a "fake Haribo". By using (13) we obtain that we have something "fake", something which is in the intersection between "fake" and "gummy bear". This should not be mistaken by a "fake gummy bear"!

Theorem 2. *For all concepts D, E and atomic concepts A applies*

$$
\begin{aligned}
A \multimap D &\sqsubseteq A && \textit{Inheritance} && (13)\\
A \multimap D \sqcup A \multimap \neg D &\equiv A && \textit{Excluded Middle} && (14)\\
A \multimap (D \sqcap E) &\sqsubseteq A \multimap D \sqcap A \multimap E && \textit{Intersection} && (15)
\end{aligned}
$$

Proof.

By union (7) we have	$A \multimap D \sqcup A \multimap \neg D \equiv C \multimap (D \sqcup \neg D)$,
and from $D \sqcup \neg D \equiv \top$ follows by (12)	$\equiv A \multimap \top$
and by universality (8)	$\equiv A$.

Thus (14) and (13) are shown. (15) follows due to generalization (12), since $D \sqcap E \sqsubseteq D$ and $D \sqcap E \sqsubseteq E$.

3 Modal Logic Property

Let C be a concept. If we consider $C \multimap$ as a modal operator $\Diamond_C$ in a multi-modal logic, then the equivalent of axiom K

$$\Box_C(P \supset Q) \supset (\Box_C P \supset \Box_C Q) \tag{16}$$

$$\iff \Diamond_C(P \wedge \neg Q) \vee (\Diamond_C \neg P \vee \neg \Diamond_C \neg Q) \tag{17}$$

is provable. We can transform (17) directly into the syntax I use, which has neither classical implication nor a necessity operator:

Theorem 3 (Axiom K).

$$C \multimap (P \sqcap \neg Q) \;\sqcup\; C \multimap \neg P \;\sqcup\; \neg(C \multimap \neg Q) \;\equiv\; \top \tag{18}$$

Proof.

$$
\begin{aligned}
\top \;&\equiv\; C \multimap \neg Q \;\sqcup\; \neg(C \multimap \neg Q) && (19)\\
&\sqsubseteq\; C \multimap ((P \sqcap \neg Q) \sqcup \neg P) \;\sqcup\; \neg(C \multimap \neg Q) && (20)\\
&\equiv\; C \multimap (P \sqcap \neg Q) \;\sqcup\; C \multimap \neg P \;\sqcup\; \neg(C \multimap \neg Q) && (21)
\end{aligned}
$$

(20) follows by using the generalization rule with $\neg Q \;\sqsubseteq\; (P \sqcap \neg Q) \sqcup \neg P$.

Theorem 4 (Gödel rule).

$$\frac{D \equiv \top}{\neg(C \multimap \neg D) \equiv \top} \tag{22}$$

Proof. The Gödel rule is equivalent to

$$\frac{\neg D \equiv \bot}{C \multimap \neg D \equiv \bot} \tag{23}$$

and since $\bot$ is a subconcept of every concept:

$$\frac{\neg D \sqsubseteq \bot}{C \multimap \neg D \sqsubseteq \bot} \tag{24}$$

Axiom (9) $\bot \equiv C \multimap \bot$ results in

$$\frac{\neg D \sqsubseteq \bot}{C \multimap \neg D \sqsubseteq C \multimap \bot}. \tag{25}$$

This is a special case of the generalization rule.

4 Kripke Structure

As we learned in the previous section, the DL extension is in fact a modal logic, where concepts can be both modal operators and predicates. Now we are curious, if there is some sort of Kripke structure for a semantic representation: In this section, I will present a tentative semantics and then go on to a more formal FOL embedding.

It is possible to prove the generalization rule and some axioms in definition 1 by using axiom K and the Gödel rule. However, the universality $C \multimap \top \equiv C$ does not seem to be provable in standard modal logics, and so the most important property of inheritance is still missing. On the other hand, this is not surprising, because an adequate semantics should provide models for the complete KB, including the ABox.

For example, we want to find a model of `Glass` $\multimap$ `GummyBear`(x): Should the concept `Glass` be a modal operator and the composite concept `Glass` $\multimap$ `GummyBear` a predicate that is applied to x? Or is `GummyBear` the predicate and must be bracketed correctly as `Glass` $\multimap$ (`GummyBear`(x))? As we will see, the bracketing makes no difference.

I assume that the ABox always makes statements about the natural view, i.e. a selected current world $w^* \in W$ in the Kripke structure. Thus a glass gummy bear would be formalized in the ABox as `Glass` $\multimap$ `GummyBear`(a) in the natural view and not in a transferred view as `GummyBear`(a). The same should apply to queries like `Glass` $\multimap$ `GummyBear`. A query `GummyBear` would return no glass gummy bears.

Otherwise, the views behave like the classical semantics of DL: Each view $w \in W$ contains a subset of individuals Δ.

$$w^{\mathcal{I}} \subseteq \Delta \tag{26}$$

The TBox should hold for every view. Inheritance is created by additional limitation of the model space, so that the subset of individuals in each view is at most reduced and may never be supplemented by new individuals

$$\forall v, w \in W.\ R_C(v,w) \implies w^{\mathcal{I}} \subseteq v^{\mathcal{I}} \tag{27}$$

where R_C is the relation corresponding to the "modal operator" $C \multimap$. This is also known as shrinking domain assumption in modal logics. It takes getting used to the fact that the natural view contains all individuals; otherwise it seems reasonable that e.g. from `Glass` $\multimap$ `GummyBear`(a) also `Glass`(a) follows.

The last trick is that the modal operator $\top$ is defined as the identity relation

$$R_\top = \{(w,w) | w \in W\}, \tag{28}$$

thus universality and realness is guaranteed. The relation

$$R_\bot = \{(w, w_\bot) | w \in W\} \tag{29}$$

points to the empty view $w_\bot^{\mathcal{I}} = \emptyset$, in which there are no individuals.

The complete Kripke model can be given by the following tuple:

$$\langle W, w^*, R_{C_1}, ..., R_{C_n}, \mathcal{I} \rangle \tag{30}$$

Please note that adequacy of the calculus in definition 1 still needs to be proved relative to the semantics presented here. Currently, both calculus and semantics are still controversial.

4.1 FOL Embedding

$\mathcal{ALC}$ can be embedded into first order logic (FOL) using a translation function $\pi_x : \mathcal{L}_{\mathcal{ALC}} \to \mathcal{L}_{FOL}$, which converts $\mathcal{ALC}$ formulas into FOL with one free variable x (see [11,1])[1]. Slightly modifying π gives us the desired properties (for C, D concepts, atomic concepts A and viewpoints v_1)

$$
\begin{aligned}
{}^{v_1}\pi_x :\ \mathcal{L}^{\multimap}_{\mathcal{ALC}} &\to \mathcal{L}_{FOL} \\
C \sqcup D &\mapsto {}^{v_1}\pi_x(C) \ \vee\ {}^{v_1}\pi_x(D) && \text{(31a)} \\
\neg C &\mapsto \neg\ {}^{v_1}\pi_x(C) && \text{(31b)} \\
C \sqcap D &\mapsto {}^{v_1}\pi_x(C) \wedge\ {}^{v_1}\pi_x(D) && \text{(31c)} \\
\forall r.\ C &\mapsto \forall y\ \big(p_r(x,y) \to\ {}^{v_1}\pi_y(C)\big) && \text{(31d)} \\
\exists r.\ C &\mapsto \exists y\ \big(p_r(x,y)\ \wedge\ {}^{v_1}\pi_y(C)\big) && \text{(31e)} \\
A &\mapsto \exists v_2\ \big(q_A(v_1,v_2)\ \wedge\ e(v_2,x)\big) && \text{(31f)} \\
A \multimap D &\mapsto \exists v_2\ \big(q_A(v_1,v_2)\ \wedge\ e(v_2,x)\ \wedge\ {}^{v_2}\pi_x(D)\big) && \text{(31g)}
\end{aligned}
$$

[1] For each transformation rule, two variants are needed for the permutations of the two variables x and y. In the modified embedding presented here, the transformation rules for the permutations ${}^{v_2}\pi_x$, ${}^{v_1}\pi_y$ and ${}^{v_2}\pi_y$ can be defined likewise.

where $q_A(\cdot,\cdot)$, $p_r(\cdot,\cdot)$ and $e(\cdot,\cdot)$ are FOL-predicates. There are two main differences between this embedding and the classical embedding: First we have the extra *viewpoint* parameter v_1 and then the atomic concepts are not translated into unary predicates $A \mapsto q_A(x)$. In contrast to the classical embedding, concepts are translated into relations between viewpoints, e.g. a "fake gun" is "fake" in the designated, common sense viewpoint v^* but a gun in the faker's viewpoint.

$e(v, x)$ denotes that an individual x belongs to a viewpoint v. For example, we want to ensure that $\texttt{Glass} \multimap \texttt{GummyBear}(x)$ exists in a `Glass` and a $\texttt{Glass} \multimap \texttt{GummyBear}$ viewpoint by the shrinking domain assumption (32) for viewpoints v_1, v_2, atomic concepts A and individuals x (in correspondence to the inheritance property (13)):

$$e(v_2, x) \land q_A(v_1, v_2) \;\rightarrow\; e(v_1, x) \tag{32}$$

The relation predicates for special concepts $q_\top$ and $q_\bot$ are defined as following:

$$q_\top(v_1, v_2) \;:\leftrightarrow\; v_1 = v_2 \tag{33}$$
$$q_\bot(v_1, v_2) \;:\leftrightarrow\; \textbf{False} \tag{34}$$

5 Further Examples

Now that everything is defined and the description logic has been extended, this is the right place to sketch possible use cases.

First we have, a closer look at the biggest city as described in [7]. "Biggest city" turns out to be a relative term, since we distinguish between the biggest city in Asia $\texttt{Asian\_City} \multimap \texttt{The\_Biggest}$, the biggest city in Europe and the biggest city on earth. But how about an alternative formalization using conventional DL? It turns out that all we need would be a `greaterThan` relation between cities. For instance, the biggest city in Asia could be defined as:

$$\texttt{Biggest\_Asian\_City} \;\equiv\; \texttt{Asian\_City} \sqcap \neg\exists \texttt{greaterThan}.\, \texttt{Asian\_City} \tag{35}$$

Let us try more promising examples: Another common relative terminology is the word "normal". A prominent example of an inconsistent knowledge base is that penguins cannot fly, whereas birds can fly. This could be solved by saying that normal birds (like pigeons) fly – penguins are birds but not normal.

$$\texttt{Bird} \multimap \texttt{Normal} \sqsubseteq \texttt{Flies} \tag{36}$$
$$\texttt{Pigeon} \sqsubseteq \texttt{Bird} \multimap \texttt{Normal} \tag{37}$$
$$\texttt{Penguin} \sqsubseteq \texttt{Bird} \tag{38}$$

The next example deals with melted things, like melted ice cream, melted water ice or melted chocolate. Melted ice cream is something that you would not call ice cream, because it is no longer creamy. Melted water ice, however, is still some aggregate form of water and can become ice again. Melted chocolate remains tasty but has lost its original form forever. The TBox could be formalized as follows:

$$\texttt{Melted} \multimap \texttt{IceCream} \sqsubseteq \neg\texttt{IceCream} \tag{39}$$

$$\texttt{Melted} \multimap \texttt{WaterIce} \sqsubseteq \texttt{Water} \tag{40}$$

$$\texttt{Melted} \multimap \texttt{Chocolate} \sqsubseteq \texttt{Chocolate} \tag{41}$$

An alternative formalization of "melted" by using a time logic is conceivable, which would require another extension of DL.

Last but not least, the TBox for a vegetarian burger $\texttt{Veggie} \multimap \texttt{Burger}$:

$$\texttt{Burger} \sqsubseteq \exists\texttt{ingredient.\,Meat} \tag{42}$$

$$\texttt{Burger} \sqsubseteq \exists\texttt{ingredient.\,Bread} \tag{43}$$

$$\texttt{Veggie} \sqcap \texttt{Meat} \equiv \bot \tag{44}$$

$$\texttt{Veggie} \multimap \texttt{Bread} \sqsubseteq \texttt{Bread} \tag{45}$$

What kind of meat does a veggie burger contain? To answer this, we use the FOL embedding:

$$^{v}\pi_a(\texttt{Veggie} \multimap \texttt{Burger}) \tag{46}$$

$$=_{def} \exists v_1 \; \big(q_{\texttt{Veggie}}(v,w) \;\wedge\; e(w,x) \;\wedge\; {}^{w}\pi_a(\texttt{Burger})\big) \tag{47}$$

Axiom (42) yields:

$$\Longrightarrow \; \exists w \; \big(q_{\texttt{Veggie}}(v,w) \;\wedge\; e(w,a) \;\wedge\; {}^{w}\pi_a(\exists\texttt{ingr.\,Meat})\big) \tag{48}$$

$$=_{def} \exists w \; \big(q_{\texttt{Veggie}}(v,w) \;\wedge\; e(w,a) \;\wedge \exists y \; [p_{\texttt{ingr}}(a,y) \;\wedge\; {}^{w}\pi_y(\texttt{Meat})]\big) \tag{49}$$

$$=_{def} \exists w,y,u\big(q_{\texttt{Veggie}}(v,w) \wedge e(w,a) \wedge p_{\texttt{ingr}}(a,y) \wedge q_{\texttt{Meat}}(w,u) \wedge e(u,y)\big) \tag{50}$$

Shrinking domain assumption (32): $q_{\texttt{Meat}}(w,u) \wedge e(u,y) \rightarrow e(w,y)$

$$\Longrightarrow \; \exists w,y,u\big(p_{\texttt{ingr}}(a,y) \wedge q_{\texttt{Veggie}}(v,w) \wedge e(w,y) \wedge q_{\texttt{Meat}}(w,u) \wedge e(u,y)\big) \tag{51}$$

$$=_{def} {}^{v}\pi_a(\exists\texttt{ingr.\,Veggie} \multimap \texttt{Meat}) \tag{52}$$

Hence a veggie burger contains "veggie meat". It is easy to show that "veggie meat" cannot be "meat" using (44). Likewise, a veggie burger contains "veggie bread" (which is real bread).

6 Discussion

Language permits many paradox concepts built as a combination of incompatible terms. There are different features and mechanisms in natural language that make these paradox combinations possible: Analogies, metaphors, ambiguity, shifting definitions of terms, lexicalization and modalities are only a subset of these features. This contrasts with formal logics which are powerful because they do not possess these features, hence avoid misinterpretations. Then again, the gap between logic and language is the reason for the development of logics that reintegrate some useful language features.

In this paper, the topic is description logics only, a particular logic family for formalizing ontologies. My work was motivated on adjective-noun phrases, adjectives that can shift the meaning of the noun. A linguistic perspective on *privative adjectives* like in "stone lion", "fake gun" in formal semantics can be found in [10].

Klarman [7] had a similar approach for relative terminologies. This is connected, but independent from his later work on a framework for contextual knowledge under a multi-modal description logic, which distinguishes between object concepts and contextual concepts [8]. The formalization in [7] uses a set theoretic semantics, which satisfies the axioms in Definition 1 (except (11)) by having a different modal-operator inspired notation $\langle \texttt{Asian\_City} \rangle \texttt{The\_Biggest}$ instead of $\texttt{Asian\_City} \multimap \texttt{The\_Biggest}$. This operator is not limited to atomic concepts $\langle A \rangle$ – complex concept constructions $\langle C \rangle$ are also allowed. Set theoretic semantics brings also a controversial equivalence

$$\langle D \rangle C \sqcap \langle D \rangle \neg C \equiv \bot \tag{53}$$

quite uncommon for modal logics. This means we cannot have

$$(\texttt{Glass} \multimap \texttt{Edible} \sqcap \texttt{Glass} \multimap \neg \texttt{Edible})\ (x). \tag{54}$$

I think that (54) is reasonable in colloquial language: First, observe that every glass gummy bear is a glass model of a gummy bear, hence a glass model of something edible. Second, observe that x is a glass model and that every glass model is a glass model of something not edible.

Another approach extending description logics is by using *conceptual blending* and *combinatorial creativity* for concept invention [5,4,12]. The blending process to invent words like "houseboat" involves generalization and iterative consistency checking and evaluation. Although combinatorial creativity combines incompatible concepts too, the idea of a conceptual blending differs from the viewpoint semantics presented here. On a closer look there are many differences between the use cases of conceptual blending and adjectivized concepts: the former modifies the relational role structure for generating a generic space, the latter is independent from the role structure. The consistency in conceptual blending is a result of an optimization process with "optimality principles defined in a vague cognitive way"[5] whereas inconsistencies are avoided (but not excluded) when using adjectivized concepts. Also note that the examples differ: "houseboat" is a *dvandva*, a special form of a compound word where neither house nor boat dominates the meaning. The production (or invention) of new compound words is an infrequent morphological process. On the other side, the use of adjective noun phrases does not require a production process: It seems that everyone understands combinations like "stone lion", "veggie burger" and "fake gun" immediately.

There are other approaches for modeling that birds fly, whereas penguins don't and that gummy bears are edible whereas glass gummy bears are not edible: Defeasable rules provide such a mechanism. [2] provides defeasable description logics using defeasable subsumption and [6] solves this problem by introducing a "normal" operator $\mathtt{T}(\bullet)$. A general approach uses the KLM postulates [3].

Although $\mathcal{ALC}$ is equivalent to a propositional multi-modal logic, it is not so uncommon to extend DL with modal logic in new dimensions. Besides this work, there are several attempts, see [11] for an overview.

A new type of concept composition for description logics was presented, which was motivated by colloquial language examples. This could be useful for semantic web ontologies, where the focus is on natural and compact knowledge representations. If you have a large knowledge base, and a fixed TBox including the fact that a "gummy bear" is edible, you might not want to alter the TBox for allowing glass gummy bears, because thousands of ABox assertions depend on a fixed and exact definition. With the description logic extension presented here, there is no need to alter the TBox, you could use $\mathtt{Glass} \multimap \mathtt{GummyBear}$ even if the TBox avoids using the $\multimap$ operator.

Beside the special concepts $\bot$ and $\top$, the introduction of common generalization concepts for vagueness or falsehood is conceivable. Consider a false fireplace: This is not a real fireplace, but it can be called a fireplace in the broader sense. A "possible car" $\Diamond \multimap \mathtt{Car}$ could be a car-like vehicle that we only see from distance and therefore cannot exactly identify.

By allowing only atomic concepts $A \multimap$ on the left side of the $\multimap$ operator, we have a simple formalization with only finitely many modal operators. Complex concepts $C \multimap$ could be allowed by converting them into expressions using atomic concepts only, once a normalization procedure is introduced.

In conclusion, the DL extension is more expressive then basic DL, the veggie burger is a nice example. The scope of the approach is not yet determined. Although inspired by language, it has a clear semantics and no vagueness or ambiguity. This is a great advantage of logical formalism over natural language, if it is not the main goal to formalize natural language.

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