Discrete Tableau Algorithms for $\mathcal{FSHI}$*

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Abstract

A variety of fuzzy description logics are proposed to extend classical description logics with fuzzy capability. However, reasoning with general TBoxes is still an open problem in fuzzy description logics. In this paper, we present a novel discrete tableau algorithm for a given fuzzy description logic $\mathcal{FSHI}$ with general TBoxes, which tries to construct discrete tableaus of $\mathcal{FSHI}$ knowledge bases. We prove the equivalence of existence between discrete tableaus and models of $\mathcal{FSHI}$ knowledge bases, hence getting that the discrete tableau algorithm is a sound and complete decision procedure for $\mathcal{FSHI}$ reasoning problems with general TBoxes.

1 Introduction

Increasing demands for fuzzy knowledge representation have triggered a variety of fuzzy extensions of description logics (DLs) that make them convenient to express knowledge in fuzzy cases. Straccia adopted fuzzy interpretations to propose a representative fuzzy extension $\mathcal{FALC}$ of $\mathcal{ALC}$, and designed a tableau algorithm for acyclic TBoxes [9]. Based on $\mathcal{FALC}$, Höldobler et al proposed membership manipulator constructors to define new fuzzy concepts [2]. Fuzzy extensions of more expressive DLs like $\mathcal{ALCQ}$ and $\mathcal{SHOIN}$(D) were presented

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in [6,11], but there were no reasoning algorithms for them. The concrete domains were also introduced into fuzzy DLs with an optimized reasoning technique in $\mathcal{FALC}$(D) [10]. Stoilos et al extended Straccia's fuzzy framework into OWL, hence getting a fuzzy ontology language: Fuzzy OWL [8]. They also gave a reasoning technique to deal with ABox consistency without TBoxes.

Though the fuzzy extension of DLs has done a lot, reasoning with general TBoxes is still an open problem in fuzzy DLs. In this paper, we will propose a novel discrete tableau algorithm for satisfiability of $\mathcal{FSHI}$ knowledge bases (KBs) with general TBoxes. The remaind of this paper is organized as follows. A brief introduction to $\mathcal{FSHI}$ KBs will be given in section 2. The main theoretical foundation of our discrete tableau algorithms is the discretization of fuzzy models, which will be discussed in section 3. Following that, we will present the definition of discrete tableaus and the expansion rules of discrete tableau algorithms, and propose a sketch proof of correctness and complexity of our algorithms in section 4. Finally section 5 will conclude this paper and discuss the further work.

2 A Brief Introduction to $\mathcal{FSHI}$

$\mathcal{FSHI}$ is a complex fuzzy description logic with role hierarchy, transitive and inverse role. Let $\mathrm{N_C}$ be a set of concept names and R be a set of role names with transitive role names $\mathrm{R}^+ \subseteq \mathrm{R}$. $\mathcal{FSHI}$ roles are either role names $R \in \mathrm{R}$ or their inverse role R^-. $\mathcal{FSHI}$ concepts are inductively defined as follows:

1. For any $A \in \mathrm{N_C}$, A is a concept;
2. The top concept $\top$ and the bottom concept $\bot$ are concepts;
3. If C and D are two concepts and R is a role, the $\neg C$, $C \sqcap D$, $C \sqcup D$, $\exists R.C$ and $\forall R.C$ are concepts.

One of the main differences between fuzzy DLs and classical DLs is that fuzzy DLs adopt fuzzy interpretations. A fuzzy interpretation $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ of $\mathcal{FSHI}$ consists of a nonempty domain $\Delta^{\mathcal{I}}$ and an interpretation function $\cdot^{\mathcal{I}}$ mapping:

$$\begin{aligned}
\text{any individual name } a \quad & \text{into} \quad a^{\mathcal{I}} \in \Delta^{\mathcal{I}} \\
\text{any concept name } A \quad & \text{into} \quad A^{\mathcal{I}} : \Delta^{\mathcal{I}} \to [0,1] \\
\text{any role name } R \quad & \text{into} \quad R^{\mathcal{I}} : \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \to [0,1]
\end{aligned}$$

And for any transitive role name $R \in \mathrm{R}^+$, $\cdot^{\mathcal{I}}$ satisfies $\forall d, d' \in \Delta^{\mathcal{I}}$, $R^{\mathcal{I}}(d,d') \geq \sup_{x \in \Delta^{\mathcal{I}}}\{\min(R^{\mathcal{I}}(d,x), R^{\mathcal{I}}(x,d'))\}$. Intuitively, any concept name A is naturally interpreted as the membership degree function $A^{\mathcal{I}}$ w.r.t. $\Delta^{\mathcal{I}}$: for any element $d \in \Delta^{\mathcal{I}}$, $A^{\mathcal{I}}(d)$ shows the degree of d being an instance of the fuzzy concept A

$$
\begin{array}{rcl}
\top^{\mathcal{I}}(d) & = & 1 \\
\bot^{\mathcal{I}}(d) & = & 0 \\
(\neg C)^{\mathcal{I}}(d) & = & 1 - C^{\mathcal{I}}(d) \\
(C \sqcap D)^{\mathcal{I}}(d) & = & \min(C^{\mathcal{I}}(d), D^{\mathcal{I}}(d)) \\
(C \sqcup D)^{\mathcal{I}}(d) & = & \max(C^{\mathcal{I}}(d), D^{\mathcal{I}}(d)) \\
(\exists R.C)^{\mathcal{I}}(d) & = & \sup_{d' \in \Delta^{\mathcal{I}}}\{\min(R^{\mathcal{I}}(d,d'), C^{\mathcal{I}}(d'))\} \\
(\forall R.C)^{\mathcal{I}}(d) & = & \inf_{d' \in \Delta^{\mathcal{I}}}\{\max(1 - R^{\mathcal{I}}(d,d'), C^{\mathcal{I}}(d'))\} \\
(R^-)^{\mathcal{I}}(d,d') & = & R^{\mathcal{I}}(d',d)
\end{array}
$$

Figure 1: The Semantics of $\mathcal{FSHI}$

under the interpretation $\mathcal{I}$. Similarly for role name R. For complex concepts and inverse roles, $\cdot^{\mathcal{I}}$ satisfies the following conditions (see figure 1):

A general TBox $\mathcal{T}$ is a finite set of fuzzy general concept inclusions $C \sqsubseteq D$, where C and D are $\mathcal{FSHI}$ concepts. An interpretation $\mathcal{I}$ satisfies $C \sqsubseteq D$ iff $\forall d \in \Delta^{\mathcal{I}}, C^{\mathcal{I}}(d) \leq D^{\mathcal{I}}(d)$. $\mathcal{I}$ satisfies (is a fuzzy model of) a TBox $\mathcal{T}$ (written $\mathcal{I} \models \mathcal{T}$) iff $\mathcal{I}$ satisfies every inclusion in $\mathcal{T}$.

A RBox $\mathcal{R}$ is a finite set of fuzzy role inclusions $R \sqsubseteq P$, where R and P are $\mathcal{FSHI}$ roles. An interpretation $\mathcal{I}$ satisfies $R \sqsubseteq P$ iff $\forall d,\ d' \in \Delta^{\mathcal{I}}, R^{\mathcal{I}}(d,d') \leq P^{\mathcal{I}}(d,d')$. $\mathcal{I}$ satisfies (is a fuzzy model of) a RBox $\mathcal{R}$ (written $\mathcal{I} \models \mathcal{R}$) iff $\mathcal{I}$ satisfies every inclusion in $\mathcal{R}$. Here we introduce $\sqsubseteq^*$ as the transitive-reflexive closure of $\sqsubseteq$ on $\mathcal{R} \cup \{R^- \sqsubseteq P^- | R \sqsubseteq P \in \mathcal{R}\}$.

An ABox $\mathcal{A}$ is a finite set of fuzzy assertions $\alpha \bowtie n$, where α is an assertion: $a : C$ or $\langle a, b\rangle : R$, $\bowtie \in \{\geq, >, \leq, <\}$ and $n \in [0,1]$. $\mathcal{I}$ satisfies a fuzzy assertion $a{:}C \geq n$ iff $C^{\mathcal{I}}(a^{\mathcal{I}}) \geq n$. Similarly for other three cases: $<$, $\leq$ and $>$, and role assertions $\langle a, b\rangle : R \bowtie n$. $\mathcal{I}$ satisfies (is a fuzzy model of) an ABox $\mathcal{A}$ (written $\mathcal{I} \models \mathcal{A}$) iff $\mathcal{I}$ satisfies every fuzzy assertion in $\mathcal{A}$.

A $\mathcal{FSHI}$ KB $\mathcal{K} = \langle \mathcal{T}, \mathcal{R}, \mathcal{A}\rangle$ consists of its TBox $\mathcal{T}$, RBox $\mathcal{R}$ and ABox $\mathcal{A}$. An interpretation $\mathcal{I}$ is a fuzzy model of a KB $\mathcal{K}$ (written $\mathcal{I} \models \mathcal{K}$), iff $\mathcal{I}$ satisfies its TBox, RBox and ABox. $\mathcal{K}$ is satisfiable iff there is a fuzzy model of $\mathcal{K}$. In this paper, we will present a discrete tableau algorithm to decide satisfiability of $\mathcal{FSHI}$ KBs.

3 Discretization of Fuzzy Models

Before discussing discretization of fuzzy models, we analyze troubles of reasoning with general TBoxes in fuzzy DLs: the key question is "why reasoning technique for general TBoxes in classical DLs can not be applied in fuzzy DLs". To answer this question, we will compare the semantics of fuzzy DLs with classical DLs. In classical DL cases, an individual (a pair of individuals) completely belongs to a concept (a role) or not, that means in classical models any concept (role) will be interpreted as two-value degree functions: $\Delta^{\mathcal{I}}(\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}) \to \{0,1\}$. For any general concept inclusion $C \sqsubseteq D$ and any individual a, classical tableau algorithms nondeterministically "guess" the membership degrees n and m of a being an instance of C and D, for some $n, m \in \{0,1\}$ and $n \leq m$ [1]. However, such "guess" technique cannot be directly applied in fuzzy DLs. The main difficulty is that in fuzzy models concepts and roles are interpreted as complex membership degree functions by extending $\{0,1\}$ to $[0,1]$, hence the value of membership degree functions is continuous but not discrete. To solve this problem, we try to design a discretization step to translate fuzzy model into corresponding discrete model, in which any membership degree value belongs to a special discrete degree set. This discretization can enable similar "guess" technique applied in fuzzy DLs.

The first issue in discretization of fuzzy models is to decide the special discrete degree set S. Let us now proceed formally in the creation of S. Consider a $\mathcal{FSHI}$ KB $\mathcal{K} = \langle \mathcal{T}, \mathcal{R}, \mathcal{A} \rangle$. Let $N_{\mathcal{K}}$ be the set of degrees appearing in $\mathcal{A}$: $N_{\mathcal{K}} = \{n | \alpha \bowtie n \in \mathcal{A}\}$. From $N_{\mathcal{K}}$, we define the degree closure $DS_{\mathcal{K}}$ of $\mathcal{K}$ as: $DS_{\mathcal{K}} = \{0, 0.5, 1\} \cup N_{\mathcal{K}} \cup \{1 - n | n \in N_{\mathcal{K}}\}$ and sort $DS_{\mathcal{K}}$ in ascending order: $DS_{\mathcal{K}} = \{n_0, n_1, \ldots, n_s\}$, where for any $0 \leq i < s$, $n_i < n_{i+1}$. It is easy to prove that $n_0 = 0$ and $n_s = 1$; s is even; and for any $0 \leq i \leq s$, $n_i + n_{n-i} = 1$.

Consider a constant vector $M = [c_1, c_2, \ldots, c_{s/2}]$, where for any c_i, $0 < c_i < 1$. We define the $\bigotimes$ operation: $NS_{\mathcal{K}} = DS_{\mathcal{K}} \bigotimes M = \{m_1, m_2, \ldots, m_s\}$, where if $i \leq s/2$, $m_i = c_i \times n_{i-1} + (1 - c_i) \times n_i$, otherwise $m_i = (1 - c_{s+1-i}) \times n_{i-1} + c_{s+1-i} \times n_i$. Obviously for any $1 \leq i \leq s$, $m_i + m_{s+1-i} = 1$ and $n_{i-1} < m_i < n_i$.

Let $S = DS_{\mathcal{K}} \cup NS_{\mathcal{K}}$, we call S a discrete degree set w.r.t $\mathcal{K}$. Note that $|S| = 2s + 1 = \mathrm{O}(|N_{\mathcal{K}}|) = \mathrm{O}(|\mathcal{A}|)$. We also sort degrees of S in ascending order: $S = \{n_0, m_1, n_1, \ldots, n_{s-1}, m_s, n_s\}$. For a fuzzy model $\mathcal{I}_c$ of $\mathcal{K}$, if every membership degree value of $C^{\mathcal{I}_c}(\ )$ or $R^{\mathcal{I}_c}(\ )$ belongs to a discrete degree set S, $\mathcal{I}_c$ is called a discrete model of $\mathcal{K}$ within S. Following theorem guarantees the equivalence between existence of fuzzy models and discrete models of $\mathcal{K}$.

Theorem 1 *For any $\mathcal{K} = \langle \mathcal{T}, \mathcal{R}, \mathcal{A} \rangle$ and any discrete degree set S w.r.t $\mathcal{K}$, $\mathcal{K}$ has a fuzzy model, iff it has a discrete model within S.*

Proof.⇒) Let $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ be a fuzzy model of $\mathcal{K}$ and the degree set $S = \{n_0, m_1, n_1, \ldots, n_{s-1}, m_s, n_s\}$. Consider a translation function $\varphi(\ ) : [0,1] \to S$:

$$\varphi(x) = \begin{cases} n_i & \text{if} \quad x = n_i \\ m_i & \text{if} \quad n_{i-1} < x < n_i \end{cases}$$

Here we enumerate some properties of $\varphi(\ )$, which are useful for the following proof: for any $x \leq y$, $\varphi(x) \leq \varphi(y)$; for any $x < y$, if x or $y \in DS_{\mathcal{K}}$, $\varphi(x) < \varphi(y)$; and for any x and y, $\varphi(1 - x) = 1 - \varphi(x)$, $\varphi(\max(x, y)) = \max(\varphi(x), \varphi(y))$, and $\varphi(\min(x, y)) = \min(\varphi(x), \varphi(y))$.

Based on $\varphi(\ )$, we construct a discrete model $\mathcal{I}_c = \langle \Delta^{\mathcal{I}_c}, \cdot^{\mathcal{I}_c} \rangle$ within S from $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$:

- The interpretation domain $\Delta^{\mathcal{I}_c}$ is defined as: $\Delta^{\mathcal{I}_c} = \Delta^{\mathcal{I}}$;
- The interpretation function $\cdot^{\mathcal{I}_c}$ is defined as: for any individual name a, $a^{\mathcal{I}_c} = a^{\mathcal{I}}$; for any concept name A and any role name R: $A^{\mathcal{I}_c}(\ ) = \varphi(A^{\mathcal{I}}(\ ))$ and $R^{\mathcal{I}_c}(\ ) = \varphi(R^{\mathcal{I}}(\ ))$; and for complex concept C and inverse role R^-, their interpretation are recursively defined based on membership degree functions $A^{\mathcal{I}_c}(\ )$ and $R^{\mathcal{I}_c}(\ )$ of concept names and role names.

1. For any concept C and role R and any two elements $d, d' \in \Delta^{\mathcal{I}_c}$, we show, by induction on the structure of C and R, that $C^{\mathcal{I}_c}(d) = \varphi(C^{\mathcal{I}}(d))$ and $R^{\mathcal{I}_c}(d, d') = \varphi(R^{\mathcal{I}}(d, d'))$:

   - Case A: from the construction of $\mathcal{I}_c$, $A^{\mathcal{I}_c}(d) = \varphi(A^{\mathcal{I}}(d))$;
   - Case R: the proof is similar to case A;
   - Case R^-: from the semantics of R^- in $\mathcal{I}$ and $\mathcal{I}_c$,
   $$(R^-)^{\mathcal{I}_c}(d, d') = R^{\mathcal{I}_c}(d', d) = \varphi(R^{\mathcal{I}}(d', d)) = \varphi((R^-)^{\mathcal{I}}(d, d'))$$
   - Case $\top$: for $1 \in DS_{\mathcal{K}}$, $\top^{\mathcal{I}_c}(d) = 1 = \varphi(\top^{\mathcal{I}}(d))$;
   - Case $\bot$: the proof is similar to case $\top$;
   - Case $\neg C$: from induction, $C^{\mathcal{I}_c}(d) = \varphi(C^{\mathcal{I}}(d))$. And from $\varphi(1 - x) = 1 - \varphi(x)$, we have
   $$\begin{aligned}(\neg C)^{\mathcal{I}_c}(d) &= 1 - C^{\mathcal{I}_c}(d) = 1 - \varphi(C^{\mathcal{I}}(d)) \\ &= \varphi(1 - C^{\mathcal{I}}(d)) = \varphi((\neg C)^{\mathcal{I}}(d))\end{aligned}$$
   - Case $C \sqcap D$: from induction, $C^{\mathcal{I}_c}(d) = \varphi(C^{\mathcal{I}}(d))$ and $D^{\mathcal{I}_c}(d) = \varphi(D^{\mathcal{I}}(d))$. We can get that
   $$\begin{aligned}(C \sqcap D)^{\mathcal{I}_c}(d) &= \min(C^{\mathcal{I}_c}(d), D^{\mathcal{I}_c}(d)) \\ &= \min(\varphi(C^{\mathcal{I}}(d)), \varphi(D^{\mathcal{I}}(d))) \\ &= \varphi(\min(C^{\mathcal{I}}(d), D^{\mathcal{I}}(d))) \\ &= \varphi((C \sqcap D)^{\mathcal{I}}(d))\end{aligned}$$
   - Case $C \sqcup D$: the proof is similar to case $C \sqcap D$.
   - Case $\forall R.C$: let $f(d') = \max(1 - R^{\mathcal{I}}(d, d'), C^{\mathcal{I}}(d'))$. From definition, $(\forall R.C)^{\mathcal{I}}(d) = \inf_{d' \in \Delta^{\mathcal{I}}} f(d')$. Assume there is an element d'' with the minimal value of $f(\ )$: for any d' in $\Delta^{\mathcal{I}}$, $f(d'') \leq f(d')$[1]. Let
   $$\begin{aligned}f^*(d') &= \varphi(f(d')) \\ &= \varphi(\max(1 - R^{\mathcal{I}}(d, d'), C^{\mathcal{I}}(d'))) \\ &= \max(\varphi(1 - R^{\mathcal{I}}(d, d')), \varphi(C^{\mathcal{I}}(d'))) \\ &= \max(1 - \varphi(R^{\mathcal{I}}(d, d')), \varphi(C^{\mathcal{I}}(d'))) \\ &= \max(1 - R^{\mathcal{I}_c}(d, d'), C^{\mathcal{I}_c}(d'))\end{aligned}$$

[1] The detailed proof of this assumption is given in [5]

Obviously, for $\forall d'$ in $\Delta^{\mathcal{I}_c}$, $f^*(d'') = \varphi(f(d'')) \leq \varphi(f(d')) = f^*(d')$. Then we get

$$(\forall R.C)^{\mathcal{I}_c}(d) = \inf_{d' \in \Delta^{\mathcal{I}_c}} f^*(d') = f^*(d'') \\ = \varphi(f(d'')) = \varphi((\forall R.C)^{\mathcal{I}}(d))$$

- Case $\exists R.C$: for $\neg(\exists R.C) = \forall R.\neg C$, we can get the proof from case $\neg C$ and $\forall R.C$.

2. We show $\mathcal{I}_c$ is a fuzzy model of $\mathcal{K}$.
   - Case $R \in \mathrm{R}^+$: for $\mathcal{I}$ is a fuzzy model of $\mathcal{K}$, $\forall d, d' \in \Delta^{\mathcal{I}}$, $R^{\mathcal{I}}(d, d') \geq \sup_{x \in \Delta^{\mathcal{I}}}\{R^{\mathcal{I}}(d, x), R^{\mathcal{I}}(x, d')\}$. Therefore,

   $$R^{\mathcal{I}_c}(d, d') = \varphi(R^{\mathcal{I}}(d, d')) \\ \geq \sup_{x \in \Delta^{\mathcal{I}}}\{\min(\varphi(R^{\mathcal{I}}(d, x)), \varphi(R^{\mathcal{I}}(x, d')))\} \\ = \sup_{x \in \Delta^{\mathcal{I}_c}}\{\min(R^{\mathcal{I}_c}(d, x), R^{\mathcal{I}_c}(x, d'))\}$$

   - Case $C \sqsubseteq D \in \mathcal{T}$: for $\mathcal{I}$ is a fuzzy model of $\mathcal{K}$, $\forall d \in \Delta^{\mathcal{I}}$, $C^{\mathcal{I}}(d) \leq D^{\mathcal{I}}(d)$. And from 1, for any concept C, $C^{\mathcal{I}_c}(d) = \varphi(C^{\mathcal{I}}(d))$. Therefore, $\forall d \in \Delta^{\mathcal{I}_c}$, $C^{\mathcal{I}_c}(d) = \varphi(C^{\mathcal{I}}(d)) \leq \varphi(D^{\mathcal{I}}(d)) = D^{\mathcal{I}_c}(d)$;
   - Case $R \sqsubseteq S \in \mathcal{R}$: the proof is similar to case $C \sqsubseteq D$;
   - Case $\alpha \bowtie n \in \mathcal{A}$: here we only focus on $a{:}C \geq n$. For $C^{\mathcal{I}}(a^{\mathcal{I}}) \geq n$ and $n \in DS_{\mathcal{K}}$, we can get:

   $$C^{\mathcal{I}_c}(a^{\mathcal{I}_c}) = \varphi(C^{\mathcal{I}}(a^{\mathcal{I}})) \geq \varphi(n) = n$$

From above two points, $\mathcal{I}_c$ is a discrete model of $\mathcal{K}$ within S.

$\Leftarrow$) Let $\mathcal{I}_c$ be a discrete model of $\mathcal{K}$ within S. It is also a fuzzy model of $\mathcal{K}$.$\square$

4 Discrete Tableau Algorithms

This section will talk about discrete tableau algorithms, which try to decide the existence of discrete models of a $\mathcal{FSHI}$ KB $\mathcal{K}$ by constructing a discrete tableau. Before going into the definition of discrete tableaus, we first introduce some notations. It will be assumed that the concepts appearing in tableau algorithms are written in Negation Normal Form (NNF). And for any concept C, we use $\mathrm{nnf}(C)$ to denote its equivalent form in NNF. The set of subconcepts of a concept C is denoted as $\mathrm{sub}(C)$. For a KB $\mathcal{K}$, we define $\mathrm{sub}(\mathcal{K})$ as the union of all $\mathrm{sub}(C)$, when the concept C appears in $\mathcal{K}$. We also make two notions about roles to make the following consideration easier: we use $\mathrm{Inv}(R)$ to denote the inverse role of R and $\mathrm{Tran}(R)$ as a Boolean value to tell whether R is transitive. $\mathrm{Trans}(R)$ =True, iff R or $\mathrm{Inv}(R) \in \mathrm{R}^+$ or there is a role P with (1) $P \sqsubseteq^* R$ and $R \sqsubseteq^* P$; and (2) P or $\mathrm{Inv}(P) \in \mathrm{R}^+$. Moreover, we use the symbols $\rhd$ and $\lhd$ as two placeholders for the inequalities $\geq$, $>$ and $\leq$, $<$, and the symbols $\bowtie^-$, $\rhd^-$

Table 1: Conjugated pairs

	$\langle <, m\rangle$	$\langle \leq, m\rangle$
$\langle \geq, n\rangle$	$n \geq m$	$n > m$
$\langle >, n\rangle$	$\neg\exists n_1 \in S$ with $n < n_1 < m$	$n \geq m$

and $\lhd^-$ to denote the reflections of $\bowtie$, $\rhd$ and $\lhd$. For example, $\geq$ and $\leq$ are the reflections to each other. Finally, we define $\langle \bowtie, n\rangle$ as a degree pair. Two degree pairs $\langle \rhd, n\rangle$ and $\langle \lhd, m\rangle$ are called conjugated, iff they satisfy one of following conditions (see table 1).

Now we define the discrete tableau for $\mathcal{K}$. Let $\mathrm{R}_\mathcal{K}$ and $\mathrm{O}_\mathcal{K}$ be the sets of roles and individuals appearing in $\mathcal{K}$. A discrete tableau T for $\mathcal{K}$ within a degree set S is a quadruple: $\langle \mathcal{O}, \mathcal{L}, \mathcal{E}, \mathcal{V}\rangle$, where

- $\mathcal{O}$: a nonempty set of nodes;
- $\mathcal{L}$: $\mathcal{O} \to 2^M$, $M = \mathrm{sub}(\mathcal{K}) \times \{\geq, >, \leq, <\} \times S$;
- $\mathcal{E}$: $\mathrm{R}_\mathcal{K} \to 2^Q$, $Q = \{\mathcal{O} \times \mathcal{O}\} \times \{\geq, >, \leq, <\} \times S$;
- $\mathcal{V}$: $\mathrm{O}_\mathcal{K} \to \mathcal{O}$, maps any individual into a corresponding node in $\mathcal{O}$.

The discrete tableau has a forest-like structure, which is a collection of trees that correspond to individuals in the ABox $\mathcal{A}$. Every tree consists of nodes standing for the individuals, and edges representing the relations between two nodes (individuals). Each node d is labelled with a set $\mathcal{L}(d)$ of degree triples: $\langle C, \bowtie, n\rangle$, which denotes the membership degree of d being an instance of $C \bowtie n$. A pair of triple $\langle C, \bowtie, n\rangle$ and $\langle C, \bowtie', m\rangle$ are conjugated if $\langle \bowtie, n\rangle$ and $\langle \bowtie', m\rangle$ are conjugated. In a discrete tableau T, for any $d, d' \in \mathcal{O}$, $a, b \in \mathrm{O}_\mathcal{K}$, $C, D \in \mathrm{sub}(\mathcal{K})$ and $R \in \mathrm{R}_\mathcal{K}$, the following conditions must hold:

1. There are not two conjugated degree triples in $\mathcal{L}(d)$;
2. There are not inconsistent triples: $\langle \bot, \geq, n\rangle$ $(n > 0)$, $\langle \top, \leq, n\rangle$ $(n < 1)$, $\langle \bot, >, n\rangle$, $\langle \top, <, n\rangle$, $\langle C, >, 1\rangle$ and $\langle C, <, 0\rangle$ in $\mathcal{L}(d)$;
3. If $C \sqsubseteq D \in \mathcal{T}$, then there must be some $n \in S$ with $\langle C, \leq, n\rangle$ and $\langle D, \geq, n\rangle$ in $\mathcal{L}(d)$;
4. If $\langle C, \bowtie, n\rangle \in \mathcal{L}(d)$, then $\langle \mathrm{nnf}(\neg C), \bowtie^-, 1-n\rangle \in \mathcal{L}(d)$;
5. If $\langle C \sqcap D, \rhd, n\rangle \in \mathcal{L}(d)$, then $\langle C, \rhd, n\rangle$ and $\langle D, \rhd, n\rangle \in \mathcal{L}(d)$;
6. If $\langle C \sqcap D, \lhd, n\rangle \in \mathcal{L}(d)$, then $\langle C, \lhd, n\rangle$ or $\langle D, \lhd, n\rangle \in \mathcal{L}(d)$;
7. If $\langle C \sqcup D, \rhd, n\rangle \in \mathcal{L}(d)$, then $\langle C, \rhd, n\rangle$ or $\langle D, \rhd, n\rangle \in \mathcal{L}(d)$;
8. If $\langle C \sqcup D, \lhd, n\rangle \in \mathcal{L}(d)$, then $\langle C, \lhd, n\rangle$ and $\langle D, \lhd, n\rangle \in \mathcal{L}(d)$;
9. If $\langle \forall R.C, \rhd, n\rangle \in \mathcal{L}(d)$,$\langle\langle d, d'\rangle, \rhd', m\rangle \in \mathcal{E}(R)$, and $\langle \rhd', m\rangle$ is conjugated with $\langle \rhd^-, 1-n\rangle$, then $\langle C, \rhd, n\rangle \in \mathcal{L}(d')$;
10. If $\langle \forall R.C, \lhd, n\rangle \in \mathcal{L}(d)$, then there must be a node $d' \in \mathcal{O}$ with $\langle\langle d, d'\rangle, \lhd^-, 1-n\rangle \in \mathcal{E}(R)$ and $\langle C, \lhd, n\rangle \in \mathcal{L}(d')$;
11. If $\langle \exists R.C, \rhd, n\rangle \in \mathcal{L}(d)$, then there must be a node $d' \in \mathcal{O}$ with $\langle\langle d, d'\rangle, \rhd, n\rangle \in \mathcal{E}(R)$ and $\langle C, \rhd, n\rangle \in \mathcal{L}(d')$;

12. If $\langle \exists R.C, \lhd, n\rangle \in \mathcal{L}(d)$, $\langle\langle d, d'\rangle, \rhd', m\rangle \in \mathcal{E}(R)$, and $\langle \rhd', m\rangle$ is conjugated with $\langle \lhd, n\rangle$, then $\langle C, \lhd, n\rangle \in \mathcal{L}(d')$;
13. If $\langle \forall P.C, \rhd, n\rangle \in \mathcal{L}(d)$, $\langle\langle d, d'\rangle, \rhd', m\rangle \in \mathcal{E}(R)$ for some $R \sqsubseteq^* P$ with Trans(R) =True and $\langle \rhd', m\rangle$ is conjugated with $\langle \rhd^-, 1-n\rangle$, then $\langle \forall R.C, \rhd, n\rangle \in \mathcal{L}(d')$;
14. If $\langle \exists P.C, \lhd, n\rangle \in \mathcal{L}(d)$, $\langle\langle d, d'\rangle, \rhd', m\rangle \in \mathcal{E}(R)$ for some $R \sqsubseteq^* P$ with Trans(R) =True and $\langle \rhd', m\rangle$ is conjugated with $\langle \lhd, n\rangle$, then $\langle \exists R.C, \lhd, n\rangle \in \mathcal{L}(d')$;
15. If $\langle\langle d, d'\rangle, \bowtie, n\rangle \in \mathcal{E}(R)$, then $\langle\langle d', d\rangle, \bowtie, n\rangle \in \mathcal{E}(\text{Inv}(R))$;
16. If $\langle\langle d, d'\rangle, \rhd, n\rangle \in \mathcal{E}(R)$ and $R \sqsubseteq^* P$, then $\langle\langle d, d'\rangle, \rhd, n\rangle \in \mathcal{E}(P)$;
17. If $a : C \bowtie n \in \mathcal{A}$, then $\langle C, \bowtie, n\rangle \in \mathcal{L}(\mathcal{V}(a))$;
18. If $\langle a, b\rangle : R \bowtie n \in \mathcal{A}$, then $\langle\langle \mathcal{V}(a), \mathcal{V}(b)\rangle, \bowtie, n\rangle \in \mathcal{E}(R)$.

Discrete tableau is an extension of fuzzy tableau [7] with additional conditions (condition 3) to deal with general TBoxes. For any $C \sqsubseteq D \in \mathcal{T}$, since any membership degree in the discrete model belongs to S, for any individuals d, let $d : C = n_1$ and $d : D = n_2$, where $n_1, n_2 \in S$ and $n_1 \leq n_2$. Obviously, there must be some $n \in S$ satisfying $n_1 \leq n \leq n_2$. Then we add $\langle C, \leq, n\rangle$ and $\langle D, \geq, n\rangle$ in $\mathcal{L}(d)$. For other conditions, conditions 1 and 2 prevent tableau from containing any clash; condition 4-16 are necessary for the completeness of discrete tableaus; and condition 17-18 ensure the correctness of individual mapping function $\mathcal{V}(\ )$.

Theorem 2 *For any $\mathcal{K} =< \mathcal{T}, \mathcal{R}, \mathcal{A} >$ and any discrete degree set S w.r.t $\mathcal{K}$, $\mathcal{K}$ has a discrete model within S iff it has a discrete tableau* T *within S.*

Proof. $\Leftarrow$) Let $S = \{n_0, m_1, n_1, \ldots, n_{s-1}, m_s, n_s\}$ and $\text{T} = \langle \mathcal{O}, \mathcal{L}, \mathcal{E}, \mathcal{V}\rangle$ be a discrete tableau within S. We define a sign function $h(\ )$: $S \to \{1, 2, \ldots, 2s+1\}$. For any $0 \leq i \leq s$, $h(n_i) = 2i + 1$ and $h(m_i) = 2i$. Obviously, for any $x \in S$, x is the $h(x)$-th minimal element in S. And we define $g(\ )$ as the inverse function of $h(\ )$. Based on T, we construct a discrete model $\mathcal{I}_c = \langle \Delta^{\mathcal{I}_c}, \cdot^{\mathcal{I}_c}\rangle$ of $\mathcal{K}$ within S:

- The interpretation domain $\Delta^{\mathcal{I}_c}$ is defined as follows: $\Delta^{\mathcal{I}_c} = \mathcal{O}$;
- The interpretation function $\cdot^{\mathcal{I}_c}$ is defined as follows: for any individual a, $a^{\mathcal{I}_c} = \mathcal{V}(a)$; for any concept name A any $d \in \Delta^{\mathcal{I}_c}$:

$$A^{\mathcal{I}_c}(d) = \max\{ \quad 0, \max\{n | \langle A, \geq, n\rangle \in \mathcal{L}(d)\}, h(g(\max\{n | \langle A, >, n\rangle \in \mathcal{L}(d)\}) + 1) \quad \}$$

And for any role name R and $d, d' \in \Delta^{\mathcal{I}_c}$, let

$$R^*(d, d') = \max\{ \quad 0, \max\{n | \langle\langle d, d'\rangle, \geq, n\rangle \in \mathcal{E}(R)\}, h(g(\max\{n | \langle\langle d, d'\rangle, >, n\rangle \in \mathcal{E}(R)\}) + 1) \quad \}$$

For any $k \geq 0$, let

$$R^*_k(d, d') = \sup_{x_1, \ldots, x_k \in \Delta^{\mathcal{I}_c}} \{\min(R^*(d, x_1), R^*(x_1, x_2), \ldots, R^*(x_{k-1}, x_k), R^*(x_k, d'))\}$$

$$R^{\mathcal{I}_c}(d, d') = \begin{cases} \sup_{k \geq 0}\{R^*_k(d, d')\} & \text{if Tran}(R) = \text{True} \\ R^*(d, d') & \text{otherwise} \end{cases}$$

From above, $A^{\mathcal{I}_c}(d)$ are defined as the minimal value to satisfy constraints in both $\geq$ and $>$ cases. Note that, in order to be greater than all values in $S^* = \{n|\langle A, >, n\rangle \in \mathcal{L}(d)\}$, $A^{\mathcal{I}_c}(d)$ must be greater than or equal to the subsequence value $h(g(\max S^*) + 1)$ of the maximal value of S^* in S. And similarly for $R^{\mathcal{I}_c}(d, d')$. And for any complex concept C and inverse role R^-, their interpretation are recursively defined based on membership degree functions $A^{\mathcal{I}_c}(\ )$ and $R^{\mathcal{I}_c}(\ )$ of concept names and role names.

1. We show, for any C and d with $\langle C, \bowtie, n\rangle \in \mathcal{L}(d)$, $C^{\mathcal{I}_c}(d) \bowtie n$.
   - Case A: from the definition of $A^{\mathcal{I}_c}(\ )$, for any $\langle A, \rhd, n\rangle \in \mathcal{L}(d)$, obviously $A^{\mathcal{I}_c}(d) \rhd n$. And for any $\langle A, \lhd, n\rangle \in \mathcal{L}(d)$, here we only focus on $\leq$ cases. Assume $A^{\mathcal{I}_c}(d) > n$, (1) if $A^{\mathcal{I}_c}(d)$ is $\max\{n|\langle A, \geq, n\rangle \in \mathcal{L}(d)\}$ or $h(g(\max\{n|\langle A, >, n\rangle \in \mathcal{L}(d)\}) + 1)$, then there must be two conjugated triples in $\mathcal{L}(d)$, which is contrary to condition 1 of the discrete tableaus; or (2) if $A^{\mathcal{I}_c}(d)$ is 0, then $n < 0$ holds which is contrary to assumption $n \in [0, 1]$.
   - Case complex concepts: the proof is similar to case A.
2. Similarly, for any R and d, d' with $\langle\langle d, d'\rangle, \bowtie, n\rangle \in \mathcal{E}(R)$, $R^{\mathcal{I}_c}(d, d') \bowtie n$.
3. We show $\mathcal{I}_c$ is a fuzzy model of $\mathcal{K}$.
   - Case $R \in \mathrm{R}^+$: from the construction of $\mathcal{I}_c$, for any $d, d' \in \Delta^{\mathcal{I}_c}$,
   $$R^{\mathcal{I}_c}(d, d') \geq \sup_{x \in \Delta^{\mathcal{I}_c}}\{R^{\mathcal{I}_c}(d, x), R^{\mathcal{I}_c}(x, d')\}$$
   - Case $C \sqsubseteq D \in \mathcal{T}$: from condition 3 of discrete tableaus, for any $d \in \Delta^{\mathcal{I}_c}$ there must be some $n \in S$ with $\langle C, \leq, n\rangle$ and $\langle D, \geq, n\rangle$ in $\mathcal{L}(d)$. And from 1, $C^{\mathcal{I}_c}(d) \leq n$ and $D^{\mathcal{I}_c}(d) \geq n$ hold. Therefore, $\mathcal{I}_c$ satisfies $C \sqsubseteq D$
   - Case $R \sqsubseteq S \in \mathcal{R}$: the proof is similar to case $C \sqsubseteq D$;
   - Case $\alpha \bowtie n \in \mathcal{A}$: here we only focus on $a : C \bowtie n$. From condition 17 of discrete tableau, $\langle C, \bowtie, n\rangle \in \mathcal{L}(\mathcal{V}(a))$. And from $a^{\mathcal{I}_c} = \mathcal{V}(a)$ and 1, we get $C^{\mathcal{I}_c}(a^{\mathcal{I}_c}) \bowtie n$.

$\Rightarrow$) Let $\mathcal{I}_c = \langle \Delta^{\mathcal{I}_c}, \cdot^{\mathcal{I}_c}\rangle$ be a discrete model within S. And we construct a discrete tableau $\mathrm{T} = \langle \mathcal{O}, \mathcal{L}, \mathcal{E}, \mathcal{V}\rangle$ from $\mathcal{I}_c$:

- $\mathcal{O}$: $\mathcal{O} = \Delta^{\mathcal{I}_c}$;
- $\mathcal{L}$: for any $d \in \mathcal{O}$, $\mathcal{L}(d) = \{\langle C, \bowtie, n\rangle | C^{\mathcal{I}_c}(d) \bowtie n, n \in S\}$;
- for any $R \in \mathrm{R}_{\mathcal{K}}$, $\mathcal{E}(R) = \{\langle\langle d, d'\rangle, \bowtie, n\rangle | R^{\mathcal{I}_c}(d, d') \bowtie n, n \in S\}$;
- $\mathcal{V}$: for any $a \in \mathrm{O}_{\mathcal{K}}$, $\mathcal{V}(a) = a^{\mathcal{I}_c}$.

From definition, $\langle C, \bowtie, n\rangle \in \mathcal{L}(d) \Leftrightarrow C^{\mathcal{I}_c}(d) \bowtie n$ and $\langle\langle d, d'\rangle, \bowtie, n\rangle \in \mathcal{E}(R) \Leftrightarrow R^{\mathcal{I}_c}(d, d') \bowtie n$. We show T is a discrete tableau of $\mathcal{K}$ within S: here we give the proof of that T satisfies condtion 3 and 4:

3. if $C \sqsubseteq D \in \mathcal{T}$, for any d, $C^{\mathcal{I}_c}(d) \leq D^{\mathcal{I}_c}(d)$. Let $C^{\mathcal{I}_c}(d) = n$ and obviously $n \in S$. From the construction of T, $\langle C, \leq, n\rangle$ and $\langle D, \geq, n\rangle \in \mathcal{L}(d)$.
4. if $\langle C, \bowtie, n\rangle \in \mathcal{L}(d)$, then $C^{\mathcal{I}_c}(d) \bowtie n$. And for the semantics of negation, $\text{nnf}(\neg C)^{\mathcal{I}_c}(d) \bowtie^- 1-n$, then $\langle \text{nnf}(\neg C), \bowtie^-, 1-n\rangle \in \mathcal{L}(d)$.

From above, T is a discrete tableau of $\mathcal{K}$ within S. □

From theorem 1 and 2, an algorithm that constructs a discrete tableau of $\mathcal{K}$ within S can be considered as a decision procedure for the satisfiability of $\mathcal{K}$. The discrete tableau algorithm works on a completion forest $\mathcal{F}_\mathcal{K}$, where each node x is labelled with $\mathcal{L}(x) \subseteq M = \text{sub}(\mathcal{K}) \times \{\geq, >, \leq, <\} \times S$; and each edge $\langle x, y\rangle$ is labelled $\mathcal{L}(\langle x, y\rangle)=\{\langle R, \bowtie, n\rangle\}$, for some $R \in \text{R}_\mathcal{K}$ and $n \in S$.

The tableau algorithm initializes $\mathcal{F}_\mathcal{K}$ to contain a root node x_a for each individual a in $\text{O}_\mathcal{K}$ and labels x_a with $\mathcal{L}(x_a) = \{\langle C, \bowtie, n\rangle | a : C \bowtie n \in \mathcal{A}\}$. Moreover, for any pair $\langle x_a, x_b\rangle$, $\mathcal{L}(\langle x_a, x_b\rangle) = \{\langle R, \bowtie, n\rangle | \langle a, b\rangle : R \bowtie n \in \mathcal{A}\}$. The algorithm expands the forest $\mathcal{F}_\mathcal{K}$ either by extending $\mathcal{L}(x)$ for the current node x or by adding new leaf node y with expansion rules in table 2.

In table 2, we adopt an optimized way to reduce "$\lhd$ rules": for any "$\lhd$" triple $\langle C, \lhd, n\rangle \in \mathcal{L}(x)$, we use $\neg^{\bowtie}$ rules to add its equivalence $\langle \text{nnf}(C), \lhd^-, 1-n\rangle$ to $\mathcal{L}(x)$, and then deal it with $\rhd$ rules.

Edges and nodes are added when expanding triples $\langle \exists R.C, \rhd, n\rangle$, $\langle \geq pR, \rhd, n\rangle$ in $\mathcal{L}(x)$. A node y is called a R-successor of another node x and x is called a R-predecessor of y, if $\langle R, \bowtie, n\rangle \in \mathcal{L}(\langle x, y\rangle)$. Ancestor is the transitive closure of predecessor. And for any two connected nodes x and y, we define $D_R(x, y)=\{\langle \rhd, n\rangle | P \sqsubseteq^* R, \langle P, \rhd, n\rangle \in \mathcal{L}(\langle x, y\rangle) \text{ or } \langle \text{Inv}(P), \rhd, n\rangle \in \mathcal{L}(\langle y, x\rangle)\} \bigcup \{\langle \lhd, n\rangle | R \sqsubseteq^* P, \langle P, \lhd, n\rangle \in \mathcal{L}(\langle x, y\rangle) \text{ or } \langle \text{Inv}(P), \lhd, n\rangle \in \mathcal{L}(\langle y, x\rangle)\}$. If $D_R(x, y) \neq \emptyset$, y is called a R-neighbor of x. As inverse role is allowed in $\mathcal{FSHI}$, we make use of dynamic blocking technique [3] to ensure the termination and correctness of our tableau algorithm. A node x is directly blocked by its ancestor y iff x is not a root node and $\mathcal{L}(x) = \mathcal{L}(y)$. A node x is indirectly blocked if its predecessor is blocked. A node x is blocked iff it is either directly or indirectly blocked.

A completion forest $\mathcal{F}_\mathcal{K}$ is said to contain a clash, if for a node x in $\mathcal{F}_\mathcal{K}$, $\mathcal{L}(x)$ contains two conjugated triples or an inconsistent triple (see condition 2 of discrete tableaus). A completion forest $\mathcal{F}_\mathcal{K}$ is clash-free if it does not contain any clash, and it is complete if none of the expansion rules are applicable.

From dynamic blocking technique, the worst-case complexity of our tableau algorithm is 2NEXPTIME [4]. And the soundness and completeness of our tableau algorithm are guaranteed by the following theorem.

Table 2: Expansion rules of discrete Tableau

Rule name	Description
KB rule:	if $C \sqsubseteq D \in \mathcal{T}$ and there is no n with $\langle C, \leq, n\rangle$ and $\langle D, \geq, n\rangle$ in $\mathcal{L}(x)$; then $\mathcal{L}(x) \to \mathcal{L}(x) \cup \{\langle C, \leq, n\rangle, \langle D, \geq, n\rangle\}$ for some $n \in S$.
The following rules are applied to nodes x which is not indirectly blocked.	
$\neg^{\bowtie}$ rule:	if $\langle C, \bowtie, n\rangle \in \mathcal{L}(x)$ and $\langle \mathrm{nnf}(\neg C), \bowtie^-, n\rangle \notin \mathcal{L}(x)$; then $\mathcal{L}(x) \to \mathcal{L}(x) \cup \{\langle \mathrm{nnf}(\neg C), \bowtie^-, n\rangle\}$.
$\sqcap^{\rhd}$ rule:	if $\langle C \sqcap D, \rhd, n\rangle \in \mathcal{L}(x)$, and $\langle C, \rhd, n\rangle$ or $\langle D, \rhd, n\rangle \notin \mathcal{L}(x)$; then $\mathcal{L}(x) \to \mathcal{L}(x) \cup \{\langle C, \rhd, n\rangle, \langle D, \rhd, n\rangle\}$.
$\sqcup^{\rhd}$ rule:	if $\langle C \sqcup D, \rhd, n\rangle \in \mathcal{L}(x)$, and $\langle C, \rhd, n\rangle, \langle D, \rhd, n\rangle \notin \mathcal{L}(x)$ then $\mathcal{L}(x) \to \mathcal{L}(x) \cup \{T\}$, for some $T \in \{\langle C, \rhd, n\rangle, \langle D, \rhd, n\rangle\}$
$\forall^{\rhd}$ rule:	if $\langle \forall R.C, \rhd, n\rangle \in \mathcal{L}(x)$, there is a R-neighbor y of x with $\langle \rhd', m\rangle \in D_R(x, y)$, which is conjugated with $\langle \rhd^-, 1-n\rangle$, and $\langle C, \rhd, n\rangle \notin \mathcal{L}(y)$; then $\mathcal{L}(y) \to \mathcal{L}(y) \cup \{\langle C, \rhd, n\rangle\}$.
$\forall^{+\rhd}$ rule:	if $\langle \forall P.C, \rhd, n\rangle \in \mathcal{L}(x)$, there is a R-neighbor y of x with $R \sqsubseteq^* P$, Trans(R)=True and $\langle \rhd', m\rangle \in D_R(x, y)$, $\langle \rhd', m\rangle$ is conjugated with $\langle \rhd^-, 1-n\rangle$ $\mathcal{L}(y) \to \mathcal{L}(y) \cup \{\langle \forall R.C, \rhd, n\rangle\}$.
The following rules are applied to nodes x which is not blocked.	
$\exists^{\rhd}$ rule:	if $\langle \exists R.C, \rhd, n\rangle \in \mathcal{L}(x)$; there is not a R-neighbor y of x with $\langle \rhd, n\rangle \in D_R(x, y)$ and $\langle C, \rhd, n\rangle \in \mathcal{L}(y)$. then add a new node z with $\langle R, \rhd, n\rangle \in \mathcal{L}(\langle x, z\rangle)$ and $\langle C, \rhd, n\rangle \in \mathcal{L}(z)$.

Theorem 3 *For any $\mathcal{K} = \langle \mathcal{T}, \mathcal{R}, \mathcal{A}\rangle$ and any discrete degree set S w.r.t $\mathcal{K}$, $\mathcal{K}$ has a discrete tableau within S iff the tableau algorithm can construct a complete and clash-free completion forest.*

Proof.(Sketch) Here we only focus on $\Leftarrow$). The proof of $\Rightarrow$) is similar to the one given in [3]. Let $\mathcal{F}_{\mathcal{K}}$ a complete and clash-free completion forest. We construct a discrete tableau T = $\langle \mathcal{O}, \mathcal{L}, \mathcal{E}, \mathcal{V}\rangle$ from $\mathcal{F}_{\mathcal{K}}$:

- $\mathcal{O}$: $\mathcal{O} = \{x \mid x \text{ is a node in } \mathcal{F}_{\mathcal{K}}, \text{ and it is not blocked}\}$;
- $\mathcal{L}$: for any $x \in \mathcal{O}$, $\mathcal{L}(x)$ = the labelling set $\mathcal{L}(x)$ of nodes x in $\mathcal{F}_{\mathcal{K}}$;
- $\mathcal{E}$: for any $R \in \mathrm{R}_{\mathcal{K}}$,

$$\mathcal{E}(R) = \{\langle\langle x, y\rangle, \bowtie, n\rangle \mid \begin{array}{l} 1.\, y \text{ is } R\text{-neighbor of } x \text{ and } \langle \bowtie, n\rangle \in D_R(x, y); \textbf{or} \\ 2.\, y \text{ blocks } z, \text{ and } \langle \bowtie, n\rangle \in D_R(x, z); \textbf{or} \\ 3.\, x \text{ blocks } z, \text{ and } \langle \bowtie, n\rangle \in D_{\mathrm{Inv}(R)}(y, z) \quad \} \end{array}$$

- $\mathcal{V}$: for any $a \in \mathrm{O}_{\mathcal{K}}$, $\mathcal{V}(a)$ = the initialized node x_a of a in $\mathcal{F}_{\mathcal{K}}$.

We can follow the similar steps in theorem 2 to prove that T is a discrete tableau of $\mathcal{K}$ within S.

5 Conclusion

This paper presents the discretization technique to reduce fuzzy models of $\mathcal{FSHI}$ and proposes a discrete tableau algorithm to solve satisfiability of $\mathcal{FSHI}$ KBs with general TBoxes. This discretization technique supports a new way to achieve reasoning with general TBoxes in fuzzy DLs. We will try to extend this discretization technique in more complex fuzzy DLs and design corresponding tableau algorithms for reasoning with them. Moreover, we plan to apply it in complexity research of reasoning problems in fuzzy DLs.

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