Probabilistic ABox Reasoning: Preliminary Results

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Abstract

Most probabilistic extensions of description logics focus on the terminological apparatus. While some allow for expressing probabilistic knowledge about concept assertions, systems which can express probabilistic knowledge about role assertions have received very little attention as yet. We present a system $\mathcal{PALC}$ which allows us to express degrees of belief in concept and role assertions for individuals. We introduce syntax and semantics for $\mathcal{PALC}$ and we define the corresponding reasoning problem. An independence assumption regarding the assertions for different individuals yields additional constraints on the possible interpretations. This considerably reduces the solution space of the $\mathcal{PALC}$ reasoning problem.

1 Introduction

Nowadays, description logics (DLs) are a standard formalism for knowledge representation [1]. For many applications it is important to extend DLs such that they can handle probabilistic knowledge. Most of the probabilistic extensions of DLs focus on probabilities on terminological axioms, see for example [2, 7, 9]. Notable exceptions are [8, 5]. Jäger [8] presents a system which allows for probabilistic knowledge about concept instances. He concentrates mainly on combining probabilistic terminological knowledge with probabilistic assertional knowledge by means of cross-entropy minimization. Giugno and Lukasiewicz [5] examine probabilistic ontologies for the semantic web. There, assertions of the form $C(a)$ are expressed as inclusion axioms $\{a\} \sqsubseteq C$ and probabilistic assertions are simply probabilistic inclusion axioms. None of the approaches however allows for expressing probabilistic knowledge on role instances. Halpern [6] and Bacchus [3] define logics to reason with and about probabilities. Since they work with full first-order logic, their systems are undecidable. Moreover the independence assumption fundamental to our approach does not hold for full first-order logic.

In our work, we propose the language $\mathcal{PALC}$ as an extension to $\mathcal{ALC}$. This language allows us to express degrees of belief in both, concept and role assertions for individuals. We introduce syntax and semantics of $\mathcal{PALC}$. The reasoning problem for this system is to find the possible valuations for the probabilistic constants. The expressive power of $\mathcal{ALC}$ is relatively low when compared to first-order logic. This results in a high degree of independence regarding the assertions for different individuals. The identification of some specific sets of independent assertions yields additional constraints on the interpretation of the probabilistic constants. In particular, we have less freedom in the interpretation of assertions about value restrictions. Straccia [12] introduces similar constraints in a lattice based approach to uncertainty in DLs. However, because his approach is not based on probabilities, there is no notion of independence. The additional constraints which are implied by our independence assumption will considerably reduce the solution space of the $\mathcal{PALC}$ reasoning problem.

Let us briefly recall some standard notions of probability theory [10]. A triple (Ω, F, P) on the domain Ω is called *probability space* where Ω is a non-empty set which is called *sample space*. The set of *events* F is a σ-algebra over Ω. The *probability distribution* $\mathrm{P} : F \to \mathbb{R}$ on F is a mapping from the events to the reals such that the *probability axioms* hold:

i) $\mathrm{P}(E) \geq 0$ for all $E \in F$.

ii) $\mathrm{P}(\Omega) = 1$.

iii) $\mathrm{P}(\bigcup_{i \in I} E_i) = \sum_{i \in I} \mathrm{P}(E_i)$ for any countable sequence $(E_i)_{i \in I}$ of pairwise disjoint events.

Let (Ω, F, P) be a probability space, Ω' a set, and F' a σ-algebra on Ω'. A function $X : \Omega \to \Omega'$ is called a *random variable* (for F') if $X^{-1}(A) \in F$ for every $A \in F'$. For brevity of notation we write $\{X \in A\} := \{s \in \Omega | X(s) \in A\}$ for the inverse image of $A \in F'$ under X. We can obtain a new probability space $(\Omega', F', \mathrm{P}')$ with the probability distribution $\mathrm{P}' : \Omega' \to \mathbb{R}, \mathrm{P}'(A) = \mathrm{P}(X \in A)$. P' is called the *distribution of the random variable* X.

Let Ω_j be a set with the σ-algebra F_j for each j in a non-empty index set J, and let $X_j : \Omega \to \Omega_j$ be a random variable for F_j for each j. The σ-algebra generated by $\{X_j | j \in J\}$ is

$$\sigma(\{X_j | j \in J\}) = \sigma(\bigcup_{j \in J} \{X_j^{-1}(A_j) \in F | A_j \in F_j\}).$$

For any events $A, B \in F$ with $\mathrm{P}(B) > 0$ the *conditional probability* of A given B is defined as

$$\mathrm{P}(A|B) := \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}.$$

A is said to be *independent* from event B if $\mathrm{P}(A|B)\,\mathrm{P}(B) = \mathrm{P}(A)\,\mathrm{P}(B)$. In general $\{A_j \in F | j \in J\}$ is independent if for any finite, non-empty subset $K \subseteq J$

$$\mathrm{P}(\bigcap_{k\in K} A_k) = \prod_{k\in K} \mathrm{P}(A_k).$$

Independence of random variables reduces to independence of sets of events: $\{X_j | j \in J\}$ is independent if for any finite, non-empty subset $K \subseteq J$

$$\mathrm{P}(\bigcap_{k\in K} \{X_k \in A_k\}) = \prod_{k\in K} \mathrm{P}(X_k \in A_k) \text{ for all } A_k \in F_k.$$

2 The Language $\mathcal{PALC}$

The language $\mathcal{PALC}$ extends the syntax and semantics of $\mathcal{ALC}$ such that we can state probabilistic assertions about the extensions of concepts and roles. To achieve this we need a set of *probabilistic constants* denoted by $p_0, p_1, \ldots$ which are of a different type than the individual constants and a *probability operator* P. If C is a concept, R is a role, p_0, p_1 are probabilistic constants, and a, b, c are individual constants, then $\mathrm{P}(C(a)) \doteq p_0$ and $\mathrm{P}(R(b,c)) \doteq p_1$ are *probabilistic concept assertions* and *probabilistic role assertions*, respectively. We will use $\rho, \sigma, \ldots$ to denote probabilistic assertions and $\alpha, \beta, \ldots$ for classical $\mathcal{ALC}$ assertions. A *probabilistic ABox* $\mathcal{A}$ is a finite set of probabilistic assertions. For reasons that will become clear with Lemma 3, we restrict our ABoxes such that for any role R, individual constant a, and probabilistic constant p we have $\mathrm{P}(R(a,a)) \doteq p \notin \mathcal{A}$.

The semantics of probabilistic assertions is given by a *probabilistic interpretation* $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}, \mathbf{\Omega})$ where the domain $\Delta^{\mathcal{I}}$ is a non-empty, countable set and $\mathbf{\Omega} = (\Omega, F, P)$ is a probability space. The *probabilistic interpretation function* $\cdot^{\mathcal{I}}$ interprets concepts and individuals as in $\mathcal{ALC}$ and additionally assigns a real number to each probabilistic constant.

Definition 1. Let $\sigma(\mathrm{cns}(\mathcal{PALC})^{\mathcal{I}}) \subseteq \mathcal{P}(\Delta^{\mathcal{I}})$ denote the σ-algebra generated by the set of all concepts of $\mathcal{PALC}$ interpreted by the probabilistic interpretation $\mathcal{I}$. Similarly let $\sigma(\mathrm{rls}(\mathcal{PALC})^{\mathcal{I}}) \subseteq \mathcal{P}(\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}})$ denote the the σ-algebra generated by the set of all roles of $\mathcal{PALC}$ interpreted by $\mathcal{I}$.

We associate with each individual $r \in \Delta^{\mathcal{I}}$ the random variable $X_r : \Omega \to \Delta^{\mathcal{I}}$ for $\sigma(\mathrm{cns}(\mathcal{PALC})^{\mathcal{I}})$ and with each pair of individuals $(r,s) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ the random variable $X_{rs} : \Omega \to \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ for $\sigma(\mathrm{rls}(\mathcal{PALC})^{\mathcal{I}})$. We require these random variables to be independent. That is, we require independence for the set $\{X_r | r \in \Delta^{\mathcal{I}}\} \cup \{X_{rs} | (r,s) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}\}$. For conciseness of notation we write $\mathrm{P}(r \in C^{\mathcal{I}})$ and $\mathrm{P}((r,s) \in R^{\mathcal{I}})$ instead of $\mathrm{P}(X_r \in C^{\mathcal{I}})$ and $\mathrm{P}(X_{rs} \in R^{\mathcal{I}})$, respectively. We also write $(r,s) \notin R^{\mathcal{I}}$ for $(r,s) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \setminus R^{\mathcal{I}}$. Finally we require for all $r \in \Delta^{\mathcal{I}}$, concepts C, and roles R the following condition to hold:

$$\mathrm{P}(r \in (\forall R.C)^{\mathcal{I}}) = \mathrm{P}(\bigcap_{s \in \Delta^{\mathcal{I}}} (\{(r,s) \notin R^{\mathcal{I}}\} \cup \{s \in C^{\mathcal{I}}\})). \qquad (\forall)$$

Remark 2. This condition is in allusion to the semantics in [4] and motivated by the fact that for an $\mathcal{ALC}$ interpretation $\mathcal{I}$ we have

$$r \in (\forall R.C)^{\mathcal{I}} \text{ iff } r \in \bigcap_{s \in \Delta^{\mathcal{I}}} (\{r \in \Delta^{\mathcal{I}} | (r,s) \notin R^{\mathcal{I}}\} \cup \{r \in \Delta^{\mathcal{I}} | s \in C^{\mathcal{I}}\}).$$

Assertions about concepts and roles which affect value restrictions are logically independent in $\mathcal{ALC}$ as shown by the following lemma.

Lemma 3. *Let R be a role, C ($C \not\equiv \bot, C \not\equiv \top$) a concept, and a, b_i individual constants for every $i = 1 \ldots n$. Let further $\mathcal{A} = \{R(a,b_i), C(b_i) | i = 1 \ldots n\}$. Then the $\mathcal{ALC}$ ABox $\mathcal{A}$ is independent in the sense that for every assertion $\alpha \in \mathcal{A}$ we have $\mathcal{A} \setminus \{\alpha\} \not\models \alpha$ and $\mathcal{A} \setminus \{\alpha\} \not\models \neg\alpha$.*

Remark 4. The ABox $\mathcal{A} = \{R(a,a), \forall R.\bot(a)\}$ is not independent.

A probabilistic interpretation $\mathcal{I}$ satisfies a probabilistic concept assertion $\mathrm{P}(C(a)) \doteq p_0$ iff $\mathrm{P}(a^{\mathcal{I}} \in C^{\mathcal{I}}) = {p_0}^{\mathcal{I}}$. Similarly $\mathcal{I}$ satisfies a probabilistic role assertion $\mathrm{P}(R(a,b)) \doteq p_0$ iff $\mathrm{P}((a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}}) = {p_0}^{\mathcal{I}}$. We say $\mathcal{I}$ is a *model* of a probabilistic assertion ρ if $\mathcal{I}$ satisfies ρ and write $\mathcal{I} \models \rho$. $\mathcal{I}$ satisfies a probabilistic ABox $\mathcal{A}$ iff it satisfies every element of $\mathcal{A}$. We then call $\mathcal{I}$ a model of $\mathcal{A}$ and write $\mathcal{I} \models \mathcal{A}$. For a concept C we write $\mathcal{I} \models C$ if $C^{\mathcal{I}} \neq \emptyset$.

The next lemma shows that the values assigned to concepts and roles by the probabilistic interpretation function are indeed probabilities.

Lemma 5. *Let $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}, \mathbf{\Omega})$ be a probabilistic interpretation where $\mathbf{\Omega}$ is the probability space (Ω, F, P).*

i) For each $r \in \Delta^{\mathcal{I}}$, the triple $(\Delta^{\mathcal{I}}, \sigma(\mathrm{cns}(\mathcal{PALC})^{\mathcal{I}}), \mathrm{P}_r)$ is a probability space with $\mathrm{P}_r : \Delta^{\mathcal{I}} \to \mathbb{R}$, $\mathrm{P}_r(C^{\mathcal{I}}) = \mathrm{P}(r \in C^{\mathcal{I}})$.

ii) For each $(r,s) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$, the triple $(\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}, \sigma(\mathrm{rls}(\mathcal{PALC})^{\mathcal{I}}), \mathrm{P}_{rs})$ is a probability space with $\mathrm{P}_{rs} : \Delta^{\mathcal{I}} \to \mathbb{R}$, $\mathrm{P}_{rs}(R^{\mathcal{I}}) = \mathrm{P}((r,s) \in R^{\mathcal{I}})$.

In the following we will write interpretation instead of probabilistic interpretation and similarly interpretation function instead of probabilistic interpretation function and membership function instead of probabilistic membership function if the meaning is clear from the context. For each real number $x \in \mathbb{R}$ we use its decimal representation - denoted by $\mathbf{x}$ - as a probabilistic constant and require $\mathbf{x}^{\mathcal{I}} = x$ for every interpretation $\mathcal{I}$.

We are now in the position to prove that the independence assumption and Condition $(\forall)$ imply additional constraints on the interpretation of probabilistic assertions for value restrictions.

Theorem 6. *A probabilistic interpretation $\mathcal{I}$ satisfies a probabilistic value restriction* $\mathrm{P}(\forall R.C(a)) \doteq p_0$ *iff*

$$p_0{}^{\mathcal{I}} = \prod_{s \in \Delta^{\mathcal{I}}} 1 - \mathrm{P}((a^{\mathcal{I}}, s) \in R^{\mathcal{I}})(1 - \mathrm{P}(s, C^{\mathcal{I}})).$$

Corollary 7. *A probabilistic interpretation $\mathcal{I}$ satisfies a probabilistic existential restriction* $\mathrm{P}(\exists R.C(a)) \doteq p_0$ *iff*

$$p_0{}^{\mathcal{I}} = 1 - \prod_{s \in \Delta^{\mathcal{I}}} 1 - \mathrm{P}((a^{\mathcal{I}}, s) \in R^{\mathcal{I}})\, \mathrm{P}(s \in C^{\mathcal{I}}).$$

Example 8. Consider the ABox

$$\mathcal{A} = \{\mathrm{P}(\forall R.C(a)) \doteq 1, \mathrm{P}(R(a,b)) \doteq \mathbf{x}, \mathrm{P}(C(b)) \doteq p\}$$

with $x > 0$. In absence of Condition ($\forall$) we could find a model $\mathcal{I}$ for $\mathcal{A}$ such that $0 \leq p^{\mathcal{I}} \leq 1$. Condition ($\forall$) however requires $1 \leq 1 - x(1 - p^{\mathcal{I}})$ for every interpretation $\mathcal{I}$ and thus $p^{\mathcal{I}} = 1$.

3 $\mathcal{PALC}$ Compared to $\mathcal{ALC}$

Definition 9. A function j is a *valuation (of the probabilistic constants)* of $\mathcal{PALC}$ if j maps each probabilistic constant p to a real number. Additionally, we require $j(\mathbf{x}) = x$ where $\mathbf{x}$ is the decimal representation of $x \in \mathbb{R}$. An interpretation $\mathcal{I}$ respects a valuation j, denoted by $\mathcal{I}_j$, if $\mathcal{I}$ agrees with j on the interpretation of the probabilistic constants (that is $p^{\mathcal{I}} = j(p)$ for all probabilistic constants p).

In $\mathcal{PALC}$ an assertion is assigned a probability which can vary among different models. In order to get a meaningful notion of entailment, we need to fix these probabilities.

Definition 10 (Entailment)**.** A probabilistic ABox $\mathcal{A}$ *entails* a probabilistic assertion ρ with respect to a valuation j, if every model $\mathcal{I}_j$ of $\mathcal{A}$ is also a model of ρ. We write $\mathcal{A} \models_j \rho$.

Let us restrict the semantics such that 1 is the only allowed probability value. This results in a sub-system of $\mathcal{PALC}$ which has the same expressiveness as $\mathcal{ALC}$. This sub-subsystem is sound and complete with respect to $\mathcal{ALC}$: an assertion has probability 1 in $\mathcal{PALC}$ if and only if it can be derived in $\mathcal{ALC}$.

Theorem 11. *Let $\models_{\mathcal{ALC}}$ denote the entailment relation of $\mathcal{ALC}$. Further, let α be an assertion of $\mathcal{ALC}$, $\mathcal{A}$ a probabilistic ABox containing only assertions which have probability 1, j any valuation, and $\mathcal{A}' = \{\gamma \mid \mathrm{P}(\gamma) \doteq 1 \in \mathcal{A}\}$. Then we have $\mathcal{A} \models_j \mathrm{P}(\alpha) \doteq 1$ if and only if $\mathcal{A}' \models_{\mathcal{ALC}} \alpha$.*

4 Reasoning in $\mathcal{PALC}$

In this section, we formally define the reasoning problem for $\mathcal{PALC}$. First, we define the notion of j-consistency which will be the dual of the entailment relation. We will note that in $\mathcal{PALC}$, entailment gives rather blunt results whereas j-consistency allows us to relate the probabilities of different assertions. It also conforms with the way we expect reasoning with probabilities to work.

Definition 12 (j-Consistency)**.** An ABox $\mathcal{A}$ is *j-consistent with an assertion* ρ - in symbols $\mathcal{A} \mathrel{|\!\!\approx_j} \rho$ - iff there is an interpretation $\mathcal{I}$ such that $\mathcal{I}_j \models \mathcal{A}$ and $\mathcal{I}_j \models \rho$.

Definition 13 (Negation)**.** Let ρ be a probabilistic assertion of the form $\mathrm{P}(C(a)) = p$. We call $\overline{\rho}$ the *negation* of ρ and define $\mathcal{I} \models \overline{\rho}$ iff $\mathrm{P}(a^{\mathcal{I}} \in C^{\mathcal{I}}) \neq p^{\mathcal{I}}$. The negation $\overline{\sigma}$ of a role assertion σ is defined correspondingly.

Definition 14. Let J be the set of all valuations, $\mathcal{A}$ an ABox, and ρ an assertion. We define the sets $P_{\mathcal{A},\rho} = \{j \in J | \mathcal{A} \mathrel{|\!\!\approx_j} \rho\}$ and $N_{\mathcal{A},\rho} = \{j \in J | \mathcal{A} \models_j \rho\}$.

Lemma 15. *Let $\mathcal{A}$ be an ABox and ρ an assertion. Then $P_{\mathcal{A},\rho} = J \setminus N_{\mathcal{A},\overline{\rho}}$.*

Example 16. Let $\mathcal{A} = \{\mathrm{P}(C \sqcap D(a)) \doteq p_1\}$ and $\rho = \mathrm{P}(C(a)) \doteq p_0$. Assume $\mathcal{I}_j \models \mathcal{A}$ and $\mathcal{I}_j \models \rho$. We find $0 \leq p_1^{\mathcal{I}} \leq p_0^{\mathcal{I}} \leq 1$ and thus $P_{\mathcal{A},\rho} = \{j \in J | 0 \leq j(p_1) \leq j(p_0) \leq 1\}$. Let j be such that $j(p_1) < 1$. Since, p_0 does not occur in $\mathcal{A}$, we find a model I_j of $\mathcal{A}$ with $\mathrm{P}(a^{\mathcal{I}} \in C^{\mathcal{I}}) \neq j(p_0)$ and thus $I_j \not\models_j \rho$ and $\mathcal{A} \not\models_j \rho$. We therefore find $N_{\mathcal{A},\rho} = \{j \in J | j(p_0) = j(p_1) = 1\}$.

Definition 17 ($\mathcal{PALC}$ reasoning problem)**.** Let $\mathcal{A}$ be an ABox, ρ be the probabilistic assertion $\mathrm{P}(\alpha) \doteq p_0$ where p_0 does not occur in $\mathcal{A}$, and j a valuation.

i) The triple $\langle \mathcal{A}, \rho, j \rangle$ is a called a *$\mathcal{PALC}$ reasoning problem.*

ii) A real number $s \in \mathbb{R}$ is called a *solution* to the $\mathcal{PALC}$ reasoning problem $\langle \mathcal{A}, \rho, j \rangle$ iff $\mathcal{A} \mathrel{|\!\!\approx_{j[p_0=s]}} \rho$ where

$$j[p = s](x) = \begin{cases} j(x) & \text{if } x \neq p, \\ s & \text{otherwise.} \end{cases}$$

iii) The set $S_{\langle \mathcal{A},\rho,j \rangle} = \{s \in \mathbb{R} | \mathcal{A} \mathrel{|\!\!\approx_{j[p_0=s]}} \rho\}$ is called the *set of solutions* to the $\mathcal{PALC}$ reasoning problem $\langle \mathcal{A}, \rho, j \rangle$.

Entailment and j-consistency in $\mathcal{PALC}$ can be reduced to finding the set of solutions of a $\mathcal{PALC}$ reasoning problem $\langle \mathcal{A}, \rho, j \rangle$. This is because the set of solutions $S_{\langle \mathcal{A},\rho,j \rangle}$ coincides with the set of valuations in a specific subset of $P_{\mathcal{A},\rho}$ evaluated at p_0 as the following lemma shows:

Lemma 18. *Let $P^j_{\mathcal{A},\rho} = \{j'(p_0) \in \mathbb{R} | j' \in P_{\mathcal{A},\rho}$ and $j'(p) = j(p)$ for all $p \neq p_0\}$ be the restriction of $P_{\mathcal{A},\rho}$ to j evaluated at p_0. Then $S_{\langle \mathcal{A},\rho,j\rangle} = P^j_{\mathcal{A},\rho}$.*

Example 19. Consider the ABox

$$\mathcal{A} = \{\mathrm{P}(C(b)) \doteq 1, \mathrm{P}(C(c)) \doteq 1, \mathrm{P}(R(a,b)) \doteq \mathbf{y}, \mathrm{P}(\forall R.\neg C(a)) \doteq \mathbf{x}\}$$

and $\rho = \mathrm{P}(R(a,c)) \doteq p_0$. Assume we are given an $\mathcal{I}_j$ such that $\mathcal{I}_j \models A$. Then $\mathrm{P}(b^{\mathcal{I}} \in (\neg C)^{\mathcal{I}}) = \mathrm{P}(c^{\mathcal{I}} \in (\neg C)^{\mathcal{I}}) = 0$. Together with Condition ($\forall$) we find $x + \delta = (1-y)(1-p_0^{\mathcal{I}})$ with a parameter $\delta \geq 0$. For $y = 1$ this yields the solutions $0 \leq p_0^{\mathcal{I}} \leq 1$ if $x = 0$ and no solution otherwise. For $y < 1$ we find $p_0^{\mathcal{I}} = 1 - \frac{x+\delta}{1-y}$. Therefore

$$S_{\langle \mathcal{A},\rho,j\rangle} = \begin{cases} [0, 1 - \min\{1, \frac{x}{1-y}\}] & \text{if } y < 1, \\ [0,1] & \text{if } y = 1,\ x = 0, \\ \emptyset & \text{otherwise.} \end{cases}$$

This example demonstrates that Condition ($\forall$) generally poses non-linear constraints on the possible interpretations of the probabilistic constants. A simple linear reasoning approach similar to [11] and also proposed in [1, 2] will therefore not work for $\mathcal{PALC}$. Although we have non-linear constraints on the solutions of $\mathcal{PALC}$ reasoning problems, we conjecture that the set of solutions forms a closed interval.

Conjecture 20. *The set of solutions $S_{\langle \mathcal{A},\rho,j\rangle}$ of a $\mathcal{PALC}$ reasoning problem is a closed interval. That is, there exist $p_l, p_u \in \mathbb{R}$ such that $S_{\langle \mathcal{A},\rho,j\rangle} = [p_l, p_u]$.*

Building on this, we formulate the reasoning task for $\mathcal{PALC}$ as follows:

Definition 21. Let $\langle \mathcal{A}, \rho, j\rangle$ be a $\mathcal{PALC}$ reasoning problem. The $\mathcal{PALC}$ *reasoning task* consists of finding $p_l, p_u \in \mathbb{R}$ such that $\mathcal{A} \models_{\widetilde{j[p_0=s]}} \rho$ iff $s \in [p_l, p_u]$.

5 Conclusion

$\mathcal{PALC}$ is a novel DL framework with probabilistic ABoxes. We have presented syntax and semantics for $\mathcal{PALC}$ and we have defined the corresponding reasoning problem. Previous approaches to probabilistic ABoxes only considered probabilistic concept assertions. $\mathcal{PALC}$ also allows for expressing probabilistic role assertions. Further, the identification of some specific sets of independent assertions yields additional constraints on the interpretation of probabilistic constants. This considerably reduces the solution space of the $\mathcal{PALC}$ reasoning problem. $\mathcal{PALC}$ is an extension to $\mathcal{ALC}$ if only allowing 1 as probability value.

Our primary future goal is to develop a reasoning algorithm for $\mathcal{PALC}$. We aim at a showing that the set of solutions of a $\mathcal{PALC}$ reasoning problem is a closed interval. A reasoning algorithm has then to find the lower and upper bounds of this interval. We plan to investigate the structure of the constraints which are imposed on the solution space. This should help to prove Conjecture 20. We hope that it also leads to a tractable reasoning algorithm for $\mathcal{PALC}$.

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