Towards a Prudent Argumentation Framework for Reasoning with Imperfect Ontologies

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There are several proposals to deal with inconsistencies in DL ontologies through argumentation. Different from existing approaches, in this paper, we consider the scenario that our knowledge is both uncertain and inconsistent and/or incoherent, and we propose a logic-based argumentation framework to deal with incomplete and conflicting DL ontologies. We do so by adopting a distinct notion of attack [3, 4] among arguments to encompass different forms of conflicts in DL ontologies. The paper presents the following major contributions: (1) a general framework for reasoning with uncertain, inconsistent and/or incoherent ontologies with the use of logic-based argumentation; (2) a general labelling method, sensitive to the numbers of attacks and the weights of arguments, with different interesting instantiations to identify the justification statuses of each argument; and (3) a number of inference relations derived from our framework in order to obtain meaningful answers without increasing the computational complexity of the reasoning process compared to classical DL reasoning. We also study the logical properties of these new entailment relations.

We consider, in this abstract, the possibilistic $\mathcal{ALC}$, denoted by $\mathcal{ALC}_\pi$, as an adaptation of $\mathcal{ALC}$ within a possibility theory setting [5]. A possibilistic axiom is a pair (α, w) where α is an axiom and $w \in [0,1]$ is a weight for the confidence degree of α.

We say that an $\mathcal{ALC}_\pi$ ontology $\mathcal{O}$ is conflict-free if $\mathcal{O}$ is consistent and coherent. The maximal conflict-free subontologies of $\mathcal{O}$ are defined as $MC(\mathcal{O}) = \{\mathcal{O}_1 \subseteq \mathcal{O} \mid \mathcal{O}_1 \text{ is conflict-free and } \forall\ \mathcal{O} \supseteq \mathcal{O}_2 \supset \mathcal{O}_1, \mathcal{O}_2 \text{ is not conflict-free}\}$.

Definition 1. *A prudent argument for an axiom α w.r.t. an $\mathcal{ALC}_\pi$ ontology $\mathcal{O}$ is a triple $\langle \Phi, \alpha, \omega \rangle$ such that (1) Φ is coherent, (2) Φ is a justification for α w.r.t. $\mathcal{O}_{\geq 0}$, where $\mathcal{O}_{\geq 0} = \{\alpha \mid (\alpha, \omega) \in \mathcal{O}\}$, and (3) $\omega = \min\{\omega_i \mid (\phi_i, \omega_i) \in \Phi\}$.*

Definition 2. *The argument structure for an axiom α is a pair of sets $\langle \mathcal{P}, \mathcal{S} \rangle$ with $\mathcal{P}$ the set of argumentation trees [3] for α and $\mathcal{S}$ the set of argumentation trees for $\neg\alpha$.*

We first extend some classical inference relations proposed by [2, 1] to $\mathcal{ALC}_\pi$.

Definition 3 ($\vdash^{\forall}_{\mathbf{MC}}, \vdash^{\exists}_{\mathbf{MC}}, \vdash^{no}_{\mathbf{MC}}, \vdash_A$). *Given an $\mathcal{ALC}_\pi$ ontology $\mathcal{O}$ and an axiom α:*

- *$\mathcal{O} \vdash^{\forall}_{MC} \alpha$ if $MC(\mathcal{O}) \neq \emptyset$ and for every $\mathcal{O}' \in MC(\mathcal{O})$, $\mathcal{O}' \vdash \alpha$;*
- *$\mathcal{O} \vdash^{\exists}_{MC} \alpha$ if there exists $\mathcal{O}' \in MC(\mathcal{O})$ s.t. $\mathcal{O}' \vdash \alpha$;*
- *$\mathcal{O} \vdash^{no}_{MC} \alpha$ if $\mathcal{O} \vdash^{\exists}_{MC} \alpha$ and $\forall\ \mathcal{O}' \in MC(\mathcal{O})$, $\alpha \not\bowtie \mathcal{O}'$;*
- *$\mathcal{O} \vdash_A \alpha$ if there exists an argument for α, and there is no argument for $\neg\alpha$ in $\mathcal{O}$.*

Next, we introduce several notions of consequence relations in the light of the argument structure and the labelling functions. A labelling function is to decide the state of an argumentation tree among accepted ($\mathcal{A}$), rejected ($\mathcal{R}$), or undecided ($\mathcal{U}$). Let T be an argumentation tree for an axiom α and $\mathbb{A} = \langle \Phi, \alpha, \omega \rangle$ be a prudent argument in T. A prudent argument labelling is a total function $Lab : \mathbb{A} \to \{(\mathcal{A}, \omega), (\mathcal{R}, \omega), \mathcal{U}\}$ defined as follows: (1) For $C(\mathbb{A}) = \emptyset$, $Lab(\mathbb{A}) = (\mathcal{A}, \omega)$. (2) For $C(\mathbb{A}) \neq \emptyset$, then

$$Lab(\mathbb{A}) = \begin{cases} (\mathcal{R}, a) & \text{if } a > 0 \\ (\mathcal{A}, -a) & \text{if } a < 0 \\ \mathcal{U} & \text{if } a = 0 \end{cases}$$

where $a = f(N_1, \ldots, N_m)$ for $N_{i(1\leq i\leq m)} \in C(\mathbb{A})$ is computed by a function $f : \mathbb{A}^n \mapsto [-1, 1]$.

A possible initialization of f is by the normalization of the difference between the number of the rejected children of a node and that of the accepted ones. The intuition is that if a prudent argument is attacked by more accepted defeaters than those labelled as rejected, this argument should be rejected because its defeaters are more often accepted. Otherwise, it can be accepted, unless if it has the same number of accepted and rejected defeaters where its labelling should be $\mathcal{U}$. Due to space limit, we do not initialize f in this abstract.

First, for a given argument structure $\langle \mathcal{P}, \mathcal{S}\rangle$ for α w.r.t. an $\mathcal{ALC}_\pi$ ontology $\mathcal{O}$, let us consider the following conditions:

C1. $\mathcal{P} \neq \emptyset$ and $\mathcal{S} = \emptyset$.
C2. $\exists\, T \in \mathcal{P}$, $Judge(T) = (Warranted, \omega)$.
C3. $\forall\, T \in \mathcal{P}$, $Judge(T) = (Warranted, \omega)$, and $\mathcal{P} \neq \emptyset$.
C4. $\max_{T\in\mathcal{P}}\{\omega \mid Judge(T) = (Warranted, \omega)\} \geq d$, $d \in [0, 1]$.
C5. $\forall\, T' \in \mathcal{S}$, $Judge(T') = (Unwarranted, \omega')$.
C6. $\forall\, T \in \mathcal{P}$, $Judge(T) \neq (Unwarranted, \omega)$.

Definition 4 ($\vdash_c$,$\vdash_s$)**.** *We say an axiom α is credulously (resp. skeptically) inferred from an ontology $\mathcal{O}$ with degree d, denoted $\mathcal{O} \vdash_c (\alpha, d)$ (resp. $\mathcal{O} \vdash_s (\alpha, d)$), iff. the argument structure $\langle \mathcal{P}, \mathcal{S}\rangle$ for α satisfies C1, C2, and C4 (resp. C1, C3, and C4).*

It is important to stress that the above inference relations $\vdash_A$,$\vdash_c$, and $\vdash_s$ for a conclusion α are conservative, hence rather unproductive. To relax such constraint, we propose in the following another reasoning type via three logical consequence relations, namely $\vdash^{\forall}_{\mathrm{arg}}$, $\vdash^{\exists}_{\mathrm{arg}}$ and $\vdash^{no}_{\mathrm{arg}}$.

Definition 5 ($\vdash^{\forall}_{\mathbf{arg}}$,$\vdash^{\exists}_{\mathbf{arg}}$,$\mathcal{O} \vdash^{no}_{\mathbf{arg}}$)**.** *Given an $\mathcal{ALC}_\pi$ ontology $\mathcal{O}$ and an axiom α:*

- *$\mathcal{O} \vdash^{\forall}_{arg} (\alpha, d)$ iff the argument structure $\langle \mathcal{P}, \mathcal{S}\rangle$ for α satisfies C3, C4, and C5;*
- *$\mathcal{O} \vdash^{\exists}_{arg} (\alpha, d)$ iff the argument structure $\langle \mathcal{P}, \mathcal{S}\rangle$ for α satisfies C2, C4, and C5;*
- *$\mathcal{O} \vdash^{no}_{arg} (\alpha, d)$ iff the argument structure $\langle \mathcal{P}, \mathcal{S}\rangle$ for α satisfies C2, C4, C5, C6.*

Next, we consider the following desired properties [6] of an inference relation $\vdash_x$:

- **Soundness:** If $\mathcal{O} \vdash_x (\alpha, d)$, then $\exists\ \mathcal{O}' \subseteq \mathcal{O}$ s.t. $\mathcal{O}' \nvdash_x \perp$, $\mathcal{O}' \vdash_x (\alpha, d)$, and $\mathcal{O}' \nvdash_x (\neg\alpha, d)$.
- **Consistency:** If $\mathcal{O} \vdash_x (\alpha, d)$, then $\mathcal{O} \nvdash_x (\neg\alpha, d)$.
- **Monotonicity w.r.t. degree:** If $\mathcal{O} \vdash_x (\alpha, d)$, then $\mathcal{O} \vdash_x (\alpha, d')$, where $0 \leq d' \leq d$.

Proposition 1. *We have $\vdash_c$,$\vdash_s$, $\vdash_A$, $\vdash^{\forall}_{MC}$, $\vdash^{\exists}_{MC}$, $\vdash^{no}_{MC}$, $\vdash^{\forall}_{arg}$,$\vdash^{\exists}_{arg}$, and $\vdash^{no}_{arg}$ satisfy Soundness; $\vdash_c$,$\vdash_s$, $\vdash_A$, $\vdash^{\forall}_{MC}$,$\vdash^{no}_{MC}$, $\vdash^{\forall}_{arg}$, $\vdash^{\exists}_{arg}$, and $\vdash^{no}_{arg}$ satisfy Consistency. And, $\vdash_c$,$\vdash_s$, $\vdash^{\forall}_{arg}$, $\vdash^{\exists}_{arg}$, and $\vdash^{no}_{arg}$ satisfy Monotonicity w.r.t. degree.*

Proposition 2. *The productivity comparison among the nine inference relations is given bellow. $A \Rightarrow B$ means the entailement relation A is more productive than B.*

$$\begin{tikzcd}
 & & \vdash^{\exists}_{\mathrm{MC}} & & \\
 & \vdash^{\exists}_{\mathrm{arg}} \arrow[ur, Rightarrow] & \vdash_c \arrow[u, Rightarrow] \arrow[l, Rightarrow] \arrow[r, Rightarrow] & \vdash_A \arrow[ul, Rightarrow] & \\
\vdash^{no}_{\mathrm{arg}} \arrow[ur, Rightarrow] & \vdash^{\forall}_{\mathrm{arg}} \arrow[u, Rightarrow] \arrow[l, Rightarrow] & \vdash_s \arrow[ul, Rightarrow] \arrow[u, Rightarrow] \arrow[ur, Rightarrow] \arrow[l, Rightarrow] & \vdash^{\forall}_{\mathrm{MC}} \arrow[u, Rightarrow] \arrow[l, Rightarrow] \arrow[r, Rightarrow] & \vdash^{no}_{\mathrm{MC}} \arrow[ul, Rightarrow]
\end{tikzcd}$$

Proposition 3. *For all the inference relations in the figure above, the entailment problem is in EXPTIME.*

References

1. S. Benferhat, Z. Bouraoui, M. Croitoru, O. Papini, and K. Tabia. Non-objection inference for inconsistency-tolerant query answering. In *IJCAI*, pages 3684–3690, 2016.
2. S. Benferhat, D. Dubois, and H. Prade. Argumentative inference in uncertain and inconsistent knowledge bases. In *UAI*, pages 411–419, 1993.
3. P. Besnard, É. Grégoire, and B. Raddaoui. A conditional logic-based argumentation framework. In *SUM*, pages 44–56, 2013.
4. A. Bouzeghoub, S. Jabbour, Y. Ma, and B. Raddaoui. Handling conflicts in uncertain ontologies using deductive argumentation. In *WI*, pages 65–72, 2017.
5. D. Dubois, J. Lang, and H. Prade. Possibilistic logic. In *Handbook of Logic in Artificial Intelligence and Logic Programming*, pages 439–513, 1994.
6. Z. Huang, F. van Harmelen, and A. ten Teije. Reasoning with inconsistent ontologies. In *IJCAI*, pages 454–459, 2005.