Topological Reasoning in Basic Description Logics

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1 Introduction

In the recent past, several scholars have shown interest in the development of an integration between general classification reasoning, as typically performed in Description Logic frameworks [DL2003], and Spatial Reasoning [CohnH01], usually carried out in a constraint-based context. Two approaches have been carried out in the recent past. One is based upon the extension $\mathcal{ALCRP}(\mathcal{D})$ of $\mathcal{ALC}$ with concrete domains, where the spatial reasoning capabilities of the framework are deployed by means of standard topological interpretation of rational numbers [HaLM1999]. The other one, instead, is based on the extension $\mathcal{ALCI}_{\mathcal{RCC}}$ [Wess2002] of $\mathcal{ALC}$ with role formation operators that are limited in scope to the definition of an algebraic-logic framework for spatial reasoning quite well-known in the reference literature and usually referred to as the *Region Connection Calculus* [Rand1989, Rand1992]. The goal of both these approaches is the representation of the topological properties and relationships between spatial objects that are in fact elements of the domain of the interpretation.

This work aims at representing the *topological properties of the sets of elements* that correspond to the interpretations of the concepts defined in a $\mathcal{ALC}$ terminology. In particular we introduce a rewriting operator that can be proved to behave as a Kuratowski closure. With this closure operator we define the notion of *connection* between concepts. The definition of a closure operator induces a topology [Kell1975] on the domain of the interpretation in which concepts can be open or closed and therefore they can be connected or not. The notion of connection between concepts deals with the knowledge ontology, in fact we aspect that for example the concept of *forest* will be not connected with the concept of *car*.

The introduction of rewriting operators in a description logic cannot interfere with the complexity of decision algorithms, so that if we introduce a rewriting operator in a description logic $\mathcal{L}$ that is decidable, then $\mathcal{L}$ remains decidable. However, a rewriting operator has its own computational cost, which is the complexity of applying the operator to generic descriptions of $\mathcal{L}$. The complexity of the rewriting operator is therefore worth computing.

In particular, we obtain three results. First, the extended case complexity - in which the Kuratowski operator is introduced in $\mathcal{ALC}_{\mathcal{U}}$ - is ExpTime, hence remaining decidable. The second result is that we have the same average case complexity as $\mathcal{ALC}_{\mathcal{U}}$. The third aspect is that complexity is theoretically scalable with the operators.

References

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