

Number Restrictions on Transitive Roles in Description Logics with Nominals*

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One of the main applications of description logics (DLs) is the formalization of ontologies from the biomedical domain, in which support for describing and classifying certain terms depending on the number of components (in a transitive sense) constituting them is often required, see e.g., [1]. Motivated by this type of applications, the DL community has investigated various languages supporting number restrictions on both transitive and non-transitive roles. Unfortunately, the combination of these features with other classical DL constructs, such as inverse roles or role inclusions, easily leads to undecidability [2]. On the positive side, it was shown that extensions of \mathcal{ALC} with these features (and nominals) are decidable [2, 3]. However, no (elementary) complexity bounds were known.

In this paper, we complete the picture of the complexity of the problem of concept satisfiability relative to TBoxes in DLs supporting counting over transitive roles. First, we establish a tight NEXPTIME upper bound for the DL \mathcal{SOQ} . To this aim, we provide a decomposition of \mathcal{SOQ} models, allowing to ‘independently reason’ about distinct roles. Then, based on a technique in [4], we show a small (that is, exponential) model property of each member of the decomposition, leading to the desired upper bound. For \mathcal{SON} , the restriction of \mathcal{SOQ} to unqualified number restrictions, we show that the coding of numbers has an impact on the computational complexity: satisfiability is EXPTIME-complete with unary coding of numbers, and NEXPTIME-complete with binary one. For the logics \mathcal{SHIF} and \mathcal{SHOIF} , allowing only for functionality, we show that satisfiability is not harder than when counting only over non-transitive roles is allowed. Finally, we initiate the study of *DL-Lite* with counting over transitive roles. We show that satisfiability in the *core* fragment of *DL-Lite* with role inclusions is undecidable, and provide complexity results for the case of functionality.

References

1. Wolstencroft, K., Brass, A., Horrocks, I., Lord, P., Sattler, U., Turi, D., Stevens, R.: A little semantic web goes a long way in biology. In: Proc. of ISWC-05. (2005)
2. Kazakov, Y., Sattler, U., Zolin, E.: How many legs do I have? non-simple roles in number restrictions revisited. In: Proc. of LPAR-07. (2007) 303–317
3. Kaminski, M., Smolka, G.: Terminating tableaux for \mathcal{SOQ} with number restrictions on transitive roles. In: Proc. of the 6th IFIP TC. (2010) 213–228
4. Kazakov, Y., Pratt-Hartmann, I.: A note on the complexity of the satisfiability problem for graded modal logics. In: Proc. of LICS-09. (2009) 407–416

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