Schema.org as a Description Logic

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Abstract. Schema.org is an initiative by the major search engine providers Bing, Google, Yahoo!, and Yandex that provides a collection of ontologies which webmasters can use to mark up their pages. Schema.org comes without a formal language definition and without a clear semantics. We formalize the language of Schema.org as a Description Logic (DL) and study the complexity of querying data using (unions of) conjunctive queries in the presence of ontologies formulated in this DL. In particular, we consider rewritability into FO queries and into datalog programs and investigate the possibility of classifying the data complexity of ontology-mediated queries.

1 Introduction

The Schema.org initiative was launched in 2011 and is supported today by Bing, Google, Yahoo!, and Yandex. In the spirit of the Semantic Web, it provides a collection of ontologies that establish a standard vocabulary to mark up website content with metadata about itself (https://schema.org/). In particular, web content that is generated from structured data as found in relational databases is often difficult to recover for search engines and Schema.org markup elegantly solves this problem. The markup is used by search engines to more precisely identify relevant pages, to provide richer search results, and to enable new applications. Schema.org is experiencing very rapid adoption and is used today by more than 15 million webpages including all major ones Guha [2013].

Schema.org does neither formally specify the language in which its ontologies are formulated nor does it provide a formal semantics for the published ontologies. However, the provided ontologies are extended and updated frequently and follow an underlying language pattern. This pattern and its meaning is described informally in natural language. Schema.org adopts a class-centric representation enriched with binary relations and datatypes, similar in spirit to description logics (DLs) and to the OWL family of ontology languages; the current version includes 622 classes and 891 binary relations. Partial translations into RDF and into OWL are provided by the linked data community. Based on the informal descriptions at https://schema.org/ and on the mentioned translations, Patel-Schneider [2014] develops an ontology language for Schema.org with a formal syntax and semantics that, apart from some details, can be regarded as a fragment of OWL DL.

In this paper, we abstract slightly further and view the Schema.org ontology language as a description logic, in line with the formalization by Patel-Schneider. Thus, what Schema.org calls a *type* becomes a concept name and a *property* becomes a role name. The main characteristics of the resulting 'Schema.org DL' are that (i) the language is very restricted, allowing only inclusions between concept and role names, domain and range restrictions, nominals, and datatypes; (ii) ranges and domains of roles can be restricted to *disjunctions* of concept names (possibly mixed with datatypes in range restrictions) and nominals are used in 'one-of enumerations' whose semantics also involves disjunction. While Point (i) suggests that the Schema.org DL is closely related to the tractable profiles of OWL2, because of Point (ii) it does actually not fall into any of them. There is also a close connection to the DL-Lite family of DLs Calvanese *et al.* [2007], and in particular to the DL-Lite$^{\mathcal{H}}_{\mathsf{bool}}$ variant Artale *et al.* [2009]. However, DL-Lite$^{\mathcal{H}}_{\mathsf{bool}}$ admits existential restriction, negation, conjunction, and free use of disjunction whereas the Schema.org DL allows no existential quantification and includes nominals and datatypes. We use the term *schema.org-ontology* to refer to ontologies formulated in the Schema.org language; in contrast, 'Schema.org 2015' refers to the concrete collection of ontologies provided at https://schema.org/ as of end of April, 2015.

Our main aim is to investigate the complexity of *querying data in the presence of schema.org-ontologies*, where the data is the markup that was extracted from webpages. While answering queries over such data is the main reasoning task that arises in Schema.org applications and the Schema.org initiative specifies a format for the data in terms of so-called *items*, no information at all is given on how the data is queried (or used otherwise). We consider conjunctive queries (CQs) and unions of conjunctive queries (UCQ), a basic querying mechanism that is ubiquitous in relational database systems and research, and that also can be viewed as a core of the Semantic Web query language SPARQL. In particular, we also consider CQs and UCQs without quantified variables since these are not allowed in the relevant SPARQL entailment regimes Glimm and Krötzsch [2010]. We view a pair $(\mathcal{O}, q)$ that consists of a schema.org-ontology and an actual query as a compound query called an *ontology-mediated query (OMQ)*.

We start with the observation that evaluating OMQs is intractable in general, namely Π^p_2-complete in combined complexity and coNP-complete in data complexity. In the main part of the paper, we therefore aim (i) to identify large and practically useful classes of OMQs with lower computational complexity (both combined and data complexity), and (ii) to explore the situation in much more detail to see whether we can obtain a *full classification* of each schema.org ontology or each OMQ according to its data complexity. While the utility of aim (i) is obvious, we note that aim (ii) is also most useful from a user's perspective as it clarifies the complexity of every *concrete ontology or OMQ* that might be used in an actual application. Apart from classical tractability (that is, PTime), we are particularly interested in the rewritability of OMQs into first-order (FO) queries (actually: UCQs) and into datalog programs. One reason is that this allows to implement querying based on relational database systems and datalog engines, taking advantage of those systems' efficiency and maturity. Another reason is that there is significant research on how to efficiently answer UCQs and datalog queries in cluster computing models such as MapReduce Afrati and Ullman [2011, 2012], which is rather natural when processing web-scale data.

For both aims (i) and (ii) above, we start with analyzing *basic* schema.org ontologies in which enumeration definitions ('one of' expressions) and datatypes are disallowed. Regarding aim (i), we show that all OMQs which consist of a basic schema.org-ontology and a CQ of qvar-size two (the connected components that consist exclusively of quantified variables have size at most two) are datalog-rewritable in polynomial time and can be evaluated in PTime in combined complexity. This result complements results about datalog-rewritability of OMQs for DLs with disjunction in Grau *et al.* [2013]; Kaminski *et al.* [2014b,a]. We establish the same results for OMQs that consist of an unrestricted schema.org-ontology and CQs without quantified variables.

Regarding aim (ii), we start with classifying each single schema.org-ontology $\mathcal{O}$ according to the data complexity of *all* OMQs $(\mathcal{O}, q)$ with q a UCQ. We establish a dichotomy between AC^0 and coNP in the sense that for each ontology $\mathcal{O}$ either all these OMQs are in AC^0 or there is one OMQ that is coNP-hard. The dichotomy comes with a transparent syntactic characterization and is decidable in PTIME. Though beautiful, the dichotomy is of limited use in practice since most interesting ontologies are of the intractable kind.

Therefore, we also consider an even more fine-grained classification on the level of OMQs, establishing a useful connection to constraint satisfaction problems (CSPs) in the spirit of Bienvenu *et al.* [2014b]. It turns out that even for basic schema.org-ontologies and for ontologies that consist exclusively of enumeration definitions, a complexity classification of OMQs implies a solution to the dichotomy conjecture for CSPs, which is a famous open problem Feder and Vardi [1998]; Bulatov [2011]. However, the CSP connection can also be used to obtain powerful positive results. In particular, we show that it is decidable in NEXPTIME whether an OMQ based on a schema.org-ontology and a restricted form of UCQ is FO-rewritable and, respectively, datalog-rewritable. We also establish a PSpace lower bound for this problem.

2 Preliminaries

Let $\mathsf{N_C}$, $\mathsf{N_R}$, and $\mathsf{N_I}$ be countably infinite and mutually disjoint sets of *concept names*, *role names*, and *individual names*. Throughout the paper, concepts names will be denoted by $A, B, C, \ldots$, role names by $r, s, t, \ldots$, and individual names by $a, b, c, \ldots$.

A schema.org-ontology consists of concept inclusions of different forms, role inclusions, and enumeration definitions. A *concept inclusion* takes the form $A \sqsubseteq B$ (*atomic concept inclusion*), $\mathsf{ran}(r) \sqsubseteq A_1 \sqcup \cdots \sqcup A_n$ (*range restriction*), or $\mathsf{dom}(r) \sqsubseteq A_1 \sqcup \cdots \sqcup A_n$ (*domain restriction*). A *role inclusion* takes the form $r \sqsubseteq s$.

Example 1. The following are examples of concept inclusions and role inclusions (last line) in Schema.org 2015:

$$\begin{aligned} \mathsf{Movie} &\sqsubseteq \mathsf{CreativeWork} \\ \mathsf{ran(musicBy)} &\sqsubseteq \mathsf{Person} \sqcup \mathsf{MusicGroup} \\ \mathsf{dom(musicBy)} &\sqsubseteq \mathsf{Episode} \sqcup \mathsf{Movie} \sqcup \mathsf{RadioSeries} \sqcup \mathsf{TVSeries} \\ \mathsf{sibling} &\sqsubseteq \mathsf{relatedTo} \end{aligned}$$

We now define enumeration definitions. Fix a set $\mathsf{N_E} \subseteq \mathsf{N_I}$ of *enumeration individuals* such that both $\mathsf{N_E}$ and $\mathsf{N_I} \setminus \mathsf{N_E}$ are infinite. An *enumeration definition* takes the form $A \equiv \{a_1, \ldots, a_n\}$ with $A \in \mathsf{N_C}$ and $a_1, \ldots, a_n \in \mathsf{N_E}$.

Example 2. An example of an enumeration definition in Schema.org 2015 is $\mathsf{Booktype} \equiv \{\mathsf{ebook}, \mathsf{hardcover}, \mathsf{paperback}\}$.

A *datatype* $\mathcal{D} = (D, \Delta^{\mathcal{D}})$ consists of a *datatype name* D and a non-empty set of *data values* $\Delta^{\mathcal{D}}$. Examples of datatypes in Schema.org 2015 are $\mathsf{Boolean}$, $\mathsf{Integer}$, and Text. We assume that datatype names and data values are distinct from the symbols in $\mathsf{N_C} \cup \mathsf{N_R} \cup \mathsf{N_I}$ and that there is an arbitrary but fixed set DT of datatypes such that $\Delta^{\mathcal{D}_1} \cap \Delta^{\mathcal{D}_2} = \emptyset$ for all $\mathcal{D}_1 \neq \mathcal{D}_2 \in \mathsf{DT}$.

To accommodate datatypes in ontologies, we generalize range restrictions to *range restrictions with datatypes*, which are inclusions of the form $\mathsf{ran}(r) \sqsubseteq A_1 \sqcup \cdots \sqcup A_n$ with $A_1, \dots, A_n$ concept names or datatype names from DT.

Example 3. An example of a range restriction with datatypes in Schema.org 2015 is

$$\mathsf{ran}(\mathsf{acceptsReservation}) \sqsubseteq \mathsf{Boolean} \sqcup \mathsf{Text}$$

A *schema.org-ontology* $\mathcal{O}$ is a finite set of concept inclusions (including range restrictions with datatypes), role inclusions, and enumeration definitions. We denote by $\mathsf{N_C}(\mathcal{O})$ the set of concept names in $\mathcal{O}$, by $\mathsf{N_R}(\mathcal{O})$ the set of role names in $\mathcal{O}$, and by $\mathsf{N_E}(\mathcal{O})$ the set of enumeration individuals in $\mathcal{O}$.

A *data instance* $\mathcal{A}$ is a finite set of *concept assertions* $A(a)$ where $A \in \mathsf{N_C}$ and $a \in \mathsf{N_I}$; and *role assertions* $r(a, b)$ where $r \in \mathsf{N_R}$, $a \in \mathsf{N_I}$ and $b \in \mathsf{N_I} \cup \bigcup_{\mathcal{D} \in \mathsf{DT}} \Delta^{\mathcal{D}}$. We say that $\mathcal{A}$ is a data instance *for the ontology* $\mathcal{O}$ if $\mathcal{A}$ contains no enumeration individuals except those in $\mathsf{N_E}(\mathcal{O})$. We use $\mathsf{Ind}(\mathcal{A})$ to denote the set of all individuals (including datatype elements) in $\mathcal{A}$.

Example 4. Examples for assertions are $\mathsf{Movie}(a)$, $\mathsf{name}(a, \text{'avatar'})$, $\mathsf{director}(a, b)$, $\mathsf{name}(b, \text{'Cam'})$.

Let $\mathcal{O}$ be a schema.org-ontology and $\mathcal{A}$ a data instance for $\mathcal{O}$. An *interpretation* $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ *for* $\mathcal{O}$ consists of a non-empty set $\Delta^{\mathcal{I}}$ disjoint from $\bigcup_{\mathcal{D} \in \mathsf{DT}} \Delta^{\mathcal{D}}$ and with $\Delta^{\mathcal{I}} \cap \mathsf{N_E} = \mathsf{N_E}(\mathcal{O})$, and a function $\cdot^{\mathcal{I}}$ that maps

- every concept name A to a subset $A^{\mathcal{I}}$ of $\Delta^{\mathcal{I}}$,
- every role name r to a subset $r^{\mathcal{I}}$ of $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I},\mathsf{DT}}$, where $\Delta^{\mathcal{I},\mathsf{DT}} = \Delta^{\mathcal{I}} \cup \bigcup_{\mathcal{D} \in \mathsf{DT}} \Delta^{\mathcal{D}}$;
- every individual name $a \in (\mathsf{N_I} \setminus \mathsf{N_E}) \cup \mathsf{N_E}(\mathcal{O})$ to some $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$ such that $a^{\mathcal{I}} = a$ for all $a \in \mathsf{N_E}(\mathcal{O})$.

Note that we make the standard name assumption (and, therefore, unique name assumption) for individuals in $\mathsf{N_E}$. Individual names from $\mathsf{N_E}$ that do not occur in $\mathcal{O}$ (and thus not in $\mathcal{A}$) are not interpreted by $\mathcal{I}$ to avoid enforcing infinite domains.

For an interpretation $\mathcal{I}$, set $\mathsf{dom}(r)^{\mathcal{I}} = \{d \mid (d, d') \in r^{\mathcal{I}}\}$ and $\mathsf{ran}(r)^{\mathcal{I}} = \{d' \mid (d, d') \in r^{\mathcal{I}}\}$. To achieve uniform notation, set $D^{\mathcal{I}} = \Delta^{\mathcal{D}}$ for every datatype $(D, \Delta^{\mathcal{D}})$ in DT and $d^{\mathcal{I}} = d$ for every $d \in \Delta^{\mathcal{D}}$, $\mathcal{D} \in \mathsf{DT}$. For concept or datatype names $A_1, \dots, A_n$, set $(A_1 \sqcup \cdots \sqcup A_n)^{\mathcal{I}} = A_1^{\mathcal{I}} \cup \cdots \cup A_n^{\mathcal{I}}$. An interpretation $\mathcal{I}$ for an ontology $\mathcal{O}$ *satisfies* a (concept or role) inclusion $X_1 \sqsubseteq X_2 \in \mathcal{O}$ if $X_1^{\mathcal{I}} \subseteq X_2^{\mathcal{I}}$, an enumeration definition $A \equiv \{a_1, \dots, a_n\}$ if $A^{\mathcal{I}} = \{a_1^{\mathcal{I}}, \dots, a_n^{\mathcal{I}}\}$, a concept assertion $A(a)$ if $a^{\mathcal{I}} \in A^{\mathcal{I}}$, and a role assertion $r(a, b)$ if $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$. Satisfaction of any of these objects is denoted with "$\models$", as in $\mathcal{I} \models X_1 \sqsubseteq X_2$ or $\mathcal{I} \models A(a)$.

An interpretation $\mathcal{I}$ for $\mathcal{O}$ is a *model* of $\mathcal{O}$ if it satisfies all inclusions and definitions in $\mathcal{O}$ and a *model* of a data instance $\mathcal{A}$ if it satisfies all assertions in $\mathcal{A}$. We say that $\mathcal{A}$ is *satisfiable* w.r.t. $\mathcal{O}$ if $\mathcal{O}$ and $\mathcal{A}$ have a common model. Let α be a concept or role inclusion, or an enumeration definition. We say that α *follows from* $\mathcal{O}$, in symbols $\mathcal{O} \models \alpha$, if every model of $\mathcal{O}$ satisfies α.

We introduce the query languages considered in this paper. A *term* t is either a member of $\mathsf{N_I} \cup \bigcup_{\mathcal{D} \in \mathsf{DT}} \Delta^{\mathcal{D}}$ or an *individual variable* taken from an infinite set $\mathsf{N_V}$ of such variables. A *first-order query (FOQ)* consist of a (domain-independent) first-order formula $\varphi(\boldsymbol{x})$ that uses unary predicates from $\mathsf{N_C} \cup \{D \mid (D, \mathcal{D}) \in \mathsf{DT}\}$, binary predicates from $\mathsf{N_R}$, and only terms as introduced above. The unary datatype predicates are built-ins that identify the elements of the respective datatype. We call $\boldsymbol{x}$ the *answer variables* of $\varphi(\boldsymbol{x})$, the remaining variables are called *quantified*. A query without answer variables is *Boolean*. A *conjunctive query (CQ)* is a FOQ of the form $\exists \boldsymbol{y}\, \varphi(\boldsymbol{x}, \boldsymbol{y})$ where $\varphi(\boldsymbol{x}, \boldsymbol{y})$ is a conjunction of atoms such that every answer variable x occurs in an atom that uses a symbol from $\mathsf{N_C} \cup \mathsf{N_R}$, that is, an answer variable x is not allowed to occur exclusively in atoms of the form $D(x)$ with D a datatype name (to ensure domain independence). A *union of conjunctive queries (UCQ)* is a disjunction of CQs. A CQ q can be regarded as a directed graph G^q with vertices $\{t \mid t \text{ term in } q\}$ and edges $\{(t, t') \mid r(t, t') \text{ in } q\}$. If G^q is acyclic and $r(t_1, t_2), s(t_1, t_2) \in q$ implies $r = s$, then q is an *acyclic CQ*. A UCQ is *acyclic* if all CQs in it are.

We are interested in querying data instances $\mathcal{A}$ using a UCQ $q(\boldsymbol{x})$ taking into account the knowledge provided by an ontology $\mathcal{O}$. A *certain answer to* $q(\boldsymbol{x})$ *in* $\mathcal{A}$ *under* $\mathcal{O}$ is a tuple $\boldsymbol{a}$ of elements of $\mathsf{Ind}(\mathcal{A})$ of the same length as $\boldsymbol{x}$ such that for every model $\mathcal{I}$ of $\mathcal{O}$ and $\mathcal{A}$, we have $\mathcal{I} \models q[\boldsymbol{a}]$. In this case, we write $\mathcal{O}, \mathcal{A} \models q(\boldsymbol{a})$.

Query evaluation is the problem to decide whether $\mathcal{O}, \mathcal{A} \models q(\boldsymbol{a})$. For the combined complexity of this problem, all of $\mathcal{O}, \mathcal{A}, q$, and $\boldsymbol{a}$ are the input. For the data complexity, only $\mathcal{A}$ and $\boldsymbol{a}$ are the input. It often makes sense to combine the ontology $\mathcal{O}$ and actual query $q(\boldsymbol{x})$ into an *ontology-mediated query (OMQ)* $Q = (\mathcal{O}, q(\boldsymbol{x}))$, which can be thought of as a compound overall query. The following can be shown using techniques similar to those in Eiter *et al.* [1997]; Bienvenu *et al.* [2014b].

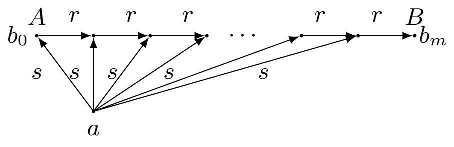
Theorem 1. *Query evaluation of CQs and UCQs under schema.org-ontologies is* Π_2^p-*complete in combined complexity. In data complexity, each OMQ* $(\mathcal{O}, q)$ *from this class can be evaluated in* coNP*; moreover, there is such a OMQ (with* q *a CQ) that is* coNP-*complete in data complexity.*

An OMQ $(\mathcal{O}, q(\boldsymbol{x}))$ is *FO-rewritable* if there exists a FOQ $Q(\boldsymbol{x})$ (called an *FO-rewriting* of $(\mathcal{O}, q(\boldsymbol{x}))$) such that for every data instance $\mathcal{A}$ for $\mathcal{O}$ and all $\boldsymbol{a} \in \mathsf{Ind}(\mathcal{A})$, we have $\mathcal{O}, \mathcal{A} \models q(\boldsymbol{a})$ iff $\mathcal{I}_{\mathcal{A}} \models Q(\boldsymbol{a})$ where $\mathcal{I}_{\mathcal{A}}$ is the interpretation that corresponds to $\mathcal{A}$ (in the obvious way).

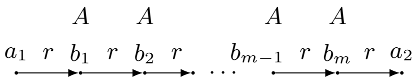
We also consider *datalog-rewritability*, defined in the same way as FO-rewritability, but using datalog programs in place of FOQs. Using Rossman's homomorphism preservation theorem Rossman [2008], one can show that an OMQ $(\mathcal{O}, q(\boldsymbol{x}))$ with $\mathcal{O}$ a schema.org-ontology and $q(\boldsymbol{x})$ a UCQ is FO-rewritable iff it has a UCQ-rewriting iff it has a non-recursive datalog rewriting, see Bienvenu *et al.* [2014b] for more details (in a slightly different context). Since non-recursive datalog-rewritings can be more succinct than UCQ-rewritings, we will generally prefer the former.

3 Basic schema.org-Ontologies

We start with considering *basic* schema.org-ontologies, which are not allowed to contain enumeration definitions and datatypes. The results obtained here can be easily extended



(a) Data instance $\mathcal{A}_m$.



A
A
A
A
a_1 r b_1 r b_2 r $\cdots$ b_{m-1} r b_m r a_2

(b) Data instance $\mathcal{A}'_m$.

Fig. 1: ABoxes used in Example 5 and the paragraph below Theorem 10.

to basic schema.org-ontologies with datatypes but do not hold for ontologies with enumeration definitions (as will be shown in the next section). In Schema.org 2015, 45 concept names from a total of 622 are defined using enumeration definitions, and hence are not covered by the results presented in this section.

We start with noting that the *entailment problem for basic schema.org-ontologies* is decidable in polynomial time. This problem is to check whether $\mathcal{O} \models \alpha$ for a given basic schema.org-ontology $\mathcal{O}$ and a given inclusion α of the form allowed in such ontologies. In fact, the algorithm is straightforward. For example, $\mathcal{O} \models \mathsf{ran}(r) \sqsubseteq A_1 \sqcup \cdots \sqcup A_n$ if there is a role name s and a range restriction $\mathsf{ran}(s) \sqsubseteq B_1 \sqcup \cdots \sqcup B_m \in \mathcal{O}$ such that $\mathcal{O}_R \models r \sqsubseteq s$ and $\mathcal{O}_C \models B_j \sqsubseteq A_1 \sqcup \cdots \sqcup A_n$ for all $1 \leq j \leq m$, where $\mathcal{O}_R$ and $\mathcal{O}_C$ denote the set of role inclusions and atomic concept inclusions in $\mathcal{O}$.

Theorem 2. *The entailment problem for basic schema.org-ontologies is in* PTIME.

The hardness results reported in Theorem 5 crucially rely on existential quantification in the actual query. In fact, it follows from results in Grau *et al.* [2013]; Kaminski *et al.* [2014b] that given an OMQ $Q = (\mathcal{O}, q(\boldsymbol{x}))$ with $\mathcal{O}$ a basic schema.org-ontology and $q(\boldsymbol{x})$ a CQ without quantified variables, it is possible to construct a non-recursive datalog rewriting of Q in polynomial time, and that such OMQs can be evaluated in PTIME in combined complexity. We aim to push this bound further by admitting restricted forms of quantification.

A CQ q has *qvar-size n* if all connected components of quantified variables in the undirected graph underlying G^q have size at most n. For example, quantifier-free CQs have qvar-size 0 and the following query $q(x, y)$ has qvar-size 1:

$$\exists z_1 \exists z_2 \bigwedge_{v \in \{x,y\}} (\mathsf{producedBy}(z_1, v) \wedge \mathsf{musicBy}(v, z_2))$$

The above consequences of the work by Grau, Kaminski, et al. can easily be extended to OMQs where queries have qvar-size one. In what follows, we consider qvar-size two, which is more subtle and where, in contrast to qvar-size one, reasoning by case distinction is required. The following example shows that there are CQs of qvar-size two for which no non-recursive datalog rewriting exists.

Example 5. Let $\mathcal{O} = \{\mathsf{ran}(s) \sqsubseteq A \sqcup B\}$ and consider the following CQ of qvar-size two: $q(x) = \exists x_1 \exists x_2 (s(x, x_1) \wedge A(x_1) \wedge r(x_1, x_2) \wedge B(x_2))$. It is easy to see that $\mathcal{O}, \mathcal{A}_m \models q(a)$ for every data instance $\mathcal{A}_m$ with $m \geq 2$ as defined in Figure 1a.

By applying locality arguments and using the data instances $\mathcal{A}_m$, one can in fact show that $(\mathcal{O}, q(x))$ is not FO-rewritable (note that removing one $r(b_i, b_{i+1})$ from $\mathcal{A}_m$ results in $q(a)$ no longer being entailed).

Theorem 3. *For every OMQ $(\mathcal{O}, q(\boldsymbol{x}))$ with $\mathcal{O}$ a basic schema.org-ontology and $q(\boldsymbol{x})$ a CQ of qvar-size at most two, one can construct a datalog-rewriting in polynomial time. Moreover, evaluating OMQs from this class is in* PTIME *in combined complexity.*

Applied to Example 5, the proof of Theorem 3 yields a datalog rewriting that consists of the rules

$$P(x_1, x_2, x) \leftarrow s(x, x_1) \land X_1(x_1) \land r(x_1, x_2) \land X_2(x_2)$$

where the X_i range over A, B, and $\exists y\, r(y, \cdot)$, plus

$$\begin{array}{ll} I_A(x_1, x) \leftarrow P(x_1, x_2, x) \land A(x_1) & I_B(x_2, x) \leftarrow P(x_1, x_2, x) \land B(x_2) \\ I_A(x_2, x) \leftarrow P(x_1, x_2, x) \land I_A(x_1, x) & I_B(x_1, x) \leftarrow P(x_1, x_2, x) \land I_B(x_2, x) \\ \mathsf{goal}(x) \leftarrow s(x, x_1) \land I_A(x_1, x) \land r(x_1, x_2) \land I_B(x_2, x). & \end{array}$$

The recursive rule for I_A (the one for I_B is dual) says that if the only option to possibly avoid a match for (x_1, x_2, x) is to color (x_1, x) with I_A, then the only way to possibly avoid a match for (x_1, x_2, x) is to color (x_2, x) with I_A (otherwise, since $\mathsf{ran}(s) \sqsubseteq A \sqcup B \in \mathcal{O}$, it would have to be colored with I_B which gives a match).

The rewriting presented in Theorem 3 can easily be extended to accommodate datatypes. For schema.org-ontologies $\mathcal{O}$ that are not basic, the rewriting is sound but not necessarily complete, and can thus be used to compute approximate query answers.

Interestingly, Theorem 3 cannot be generalized to UCQs. This follows from the result in the full version that for basic schema.org-ontologies $\mathcal{O}$ and quantifier-free UCQs $q(x)$ (even without role atoms), the problem $\mathcal{O}, \mathcal{A} \models q(a)$ is coNP-hard regarding combined complexity for data instances $\mathcal{A}$ with a single individual a. Since evaluating datalog programs in such data instances is in PTIME, datalog rewritings of UCQ-based OMQs can thus not be constructed in polynomial time (unless PTIME equals NP). We note that it is not difficult to show (and follows from FO-rewritability of instance queries in DL-Lite$^{\mathcal{H}}_{\mathsf{bool}}$ Artale *et al.* [2009]) that given an OMQ $(\mathcal{O}, q(\boldsymbol{x}))$ with $\mathcal{O}$ a basic schema.org-ontology and $q(\boldsymbol{x})$ a quantifier-free UCQ, one can construct an FO-rewriting in exponential time, and thus query evaluation is in AC^0 in data complexity.

We now classify basic schema.org-ontologies $\mathcal{O}$ according to the data complexity of evaluating OMQs $(\mathcal{O}, q)$ with q a UCQ (or CQ). It is convenient to work with *minimized* ontologies where for all inclusions $F \sqsubseteq A_1 \sqcup \cdots \sqcup A_n \in \mathcal{O}$ and all $i \leq n$, there is a model $\mathcal{I}$ of $\mathcal{O}$ and a $d \in \Delta^{\mathcal{I}}$such that d *satisfies* $F \sqcap A_i \sqcap \bigsqcap_{j \neq i} \neg A_j$ (defined in the usual way). Every schema.org-ontology can be rewritten in polynomial time into an equivalent minimized one. We establish the following dichotomy theorem.

Theorem 4. *Let $\mathcal{O}$ be a minimized basic schema.org-ontology. If there exists $F \sqsubseteq A_1 \sqcup \cdots \sqcup A_n \in \mathcal{O}$ with $n \geq 2$, then there is a Boolean CQ q that uses only concept and role names from $\mathcal{O}$ and such that $(\mathcal{O}, q)$ is* CONP*-hard in data complexity. Otherwise, a given OMQ $(\mathcal{O}, q)$ with q a UCQ can be rewritten into a non-recursive datalog-program in polynomial time (and is thus in* AC^0 *in data complexity).*

The proof of the second part of Theorem 4 is easy: if there are no $F \sqsubseteq A_1 \sqcup \cdots \sqcup A_n \in \mathcal{O}$ with $n \geq 2$, then $\mathcal{O}$ essentially is already a non-recursive datalog program and the construction is straightforward. The proof of the hardness part is obtained by extending the corresponding part of a dichotomy theorem for $\mathcal{ALC}$-ontologies of depth one Lutz and Wolter [2012]. The main differences between the two theorems are that (i) for basic schema.org-ontologies, the dichotomy is decidable in PTIME (whereas decidability is open for $\mathcal{ALC}$), (ii) the CQs in coNP-hard OMQs use only concept and role names from $\mathcal{O}$ (this is not possible in $\mathcal{ALC}$), and (iii) the dichotomy is between AC^0 and coNP whereas for $\mathcal{ALC}$ OMQs can be complete for PTIME, NL, etc.

By Theorem 4, disjunctions in domain and range restrictions are the only reason that query answering is non-tractable for basic schema.org-ontologies. In Schema.org 2015, 14% of all range restrictions and 20% of all domain restrictions contain disjunctions.

In Theorem 4, we have classified the data complexity of *ontologies*, quantifying over the actual queries. In what follows, we aim to classify the data complexity of every OMQ. This problem turns out to be much harder and, in fact, we show that a classification of the data complexity of OMQs based on basic schema.org-ontologies and UCQs implies a classification of constraint satisfaction problems according to their complexity (up to FO-reductions), a famous open problem that is the subject of significant ongoing research Feder and Vardi [1998]; Bulatov [2011].

A *signature* is a set of concept and role names (also called *symbols*). Let $\mathcal{B}$ be a finite interpretation that interprets only the symbols from a finite signature Σ. The *constraint satisfaction problem CSP*$(\mathcal{B})$ is to decide, given a data instance $\mathcal{A}$ over Σ, whether there is a homomorphism from $\mathcal{A}$ to $\mathcal{B}$. In this context, $\mathcal{B}$ is called the *template* of CSP$(\mathcal{B})$.

Theorem 5. *For every template $\mathcal{B}$, one can construct in polynomial time an OMQ $(\mathcal{O}, q)$ where $\mathcal{O}$ is a basic schema.org-ontology and q a Boolean acyclic UCQ such that the complement of CSP($\mathcal{B}$) and $(\mathcal{O}, q)$ are mutually FO-reducible.*

Theorem 13 below establishes the converse direction of Theorem 5 for unrestricted schema.org-ontologies and a large class of (acyclic) UCQs. From Theorem 13, we obtain a NEXPTIME-upper bound for deciding FO-rewritability and datalog-rewritability of a large class of OMQs. It remains open whether this bound is tight, but we can show a PSPACE lower bound for FO-rewritable using a reduction of the word problem of PSPACE Turing machines. The proof uses the ontology $\mathcal{O}$ and data instances $\mathcal{A}_m$ from Example 5 and is similar to a PSPACE lower bound proof for FO-rewritability in consistent query answering Lutz and Wolter [2015] which is, in turn, based on a construction from Cosmadakis *et al.* [1988].

Theorem 6. *It is* PSPACE*-hard to decide whether a given OMQ $(\mathcal{O}, q)$ with $\mathcal{O}$ a basic schema.org-ontology and q a Boolean acyclic UCQ is FO-rewritable.*

4 Incoherence and Unsatisfiability

In the subsequent section, we consider unrestricted schema.org ontologies instead of basic ones, that is, we add back enumeration definitions and datatypes. The purpose of this section is to deal with a complication that arises from this step, namely the potential presence of inconsistencies. We start with inconsistencies that concern the ontology alone and then consider inconsistencies that arise from combining an ontology with a data instance.

An ontology $\mathcal{O}$ is *incoherent* if there exists $X \in \mathsf{N_C} \cup \mathsf{N_R}$ such that $X^{\mathcal{I}} = \emptyset$ for all models $\mathcal{I}$ of $\mathcal{O}$. Incoherent ontologies can result from the UNA for enumeration individuals such as in the ontology $\{A \equiv \{a\}, B \equiv \{b\}, A \sqsubseteq B\}$, which has no model if $a \neq b$; they can also arise from interactions between concept names and datatypes such as in the ontology $\{\mathsf{ran}(r) \sqsubseteq \mathsf{Integer}, \mathsf{ran}(s) \sqsubseteq A, r \sqsubseteq s\}$ with $A \in \mathsf{N_C}$ which has no model $\mathcal{I}$ with $r^{\mathcal{I}} \neq \emptyset$ since $\Delta^{\mathcal{I}} \cap \Delta^{\mathsf{Integer}} = \emptyset$. Using Theorem 2, one can show:

Theorem 7. *Incoherence of schema.org-ontologies can be decided in PTime.*

We now turn to inconsistencies that arise from combining an ontology $\mathcal{O}$ with a data instance $\mathcal{A}$ for $\mathcal{O}$. As an example, consider $\mathcal{O} = \{A \equiv \{a\}, B \equiv \{b\}\}$ and $\mathcal{A} = \{A(c), B(c)\}$. Although $\mathcal{O}$ is coherent, $\mathcal{A}$ is unsatisfiable w.r.t. $\mathcal{O}$. Like incoherence, unsatisfiability is decidable in polynomial time. In fact, we can even show the following stronger result.

Theorem 8. *Given a schema.org-ontology $\mathcal{O}$, one can compute in polynomial time a non-recursive datalog program Π such that for any data instance $\mathcal{A}$ for $\mathcal{O}$, $\mathcal{A}$ is unsatisfiable w.r.t. $\mathcal{O}$ iff $\Pi(\mathcal{A}) \neq \emptyset$.*

In typical schema.org applications, the data is collected from the web and it is usually not acceptable to simply report back an inconsistency and stop processing the query. Instead, one would like to take maximum advantage of the data despite the presence of an inconsistency. There are many semantics for inconsistent query answering that can be used for this purpose. As efficiency is paramount in schema.org applications, our choice is the pragmatic intersection repair (IAR) semantics which avoids CONP-hardness in data complexity Lembo *et al.* [2010]; Rosati [2011]; Bienvenu *et al.* [2014a]. A *repair* of a data instance $\mathcal{A}$ w.r.t. an ontology $\mathcal{O}$ is a maximal subset $\mathcal{A}' \subseteq \mathcal{A}$ that is satisfiable w.r.t. $\mathcal{O}$. We use $\mathsf{rep}_{\mathcal{O}}(\mathcal{A})$ to denote the set of all repairs of $\mathcal{A}$ w.r.t. $\mathcal{O}$. The idea of IAR semantics is then to replace $\mathcal{A}$ with $\bigcap_{\mathcal{A}' \in \mathsf{rep}_{\mathcal{O}}(\mathcal{A})} \mathcal{A}'$. In other words, we have to remove from $\mathcal{A}$ all assertions that occur in some minimal subset $\mathcal{A}' \subseteq \mathcal{A}$ that is unsatisfiable w.r.t. $\mathcal{O}$. We call such an assertion a *conflict assertion*.

Theorem 9. *Given a schema.org-ontology $\mathcal{O}$ and concept name A (resp. role name r), one can compute a non-recursive datalog program Π such that for any data instance $\mathcal{A}$ for $\mathcal{O}$, $\Pi(\mathcal{A})$ is the set of all $a \in \mathsf{Ind}(\mathcal{A})$ (resp. $(a,b) \in \mathsf{Ind}(\mathcal{A})^2$) such that $A(a)$ (resp. $r(a,b)$) is a conflict assertion in $\mathcal{A}$.*

By Theorem 9, we can adopt the IAR semantics by simply removing all conflict assertions from the data instance before processing the query. Programs from Theorem 9 become exponential in the worst case, but can be expected to be very small in practical cases. In the remainder of the paper, we assume that ontologies are coherent and that $\mathcal{A}$ is satisfiable w.r.t. $\mathcal{O}$ if we query a data instance $\mathcal{A}$ using an ontology $\mathcal{O}$.

5 Unrestricted schema.org-Ontologies

We aim to lift the results from Section 3 to unrestricted schema.org-ontologies. Regarding Theorem 3, it turns out that quantified variables in CQs are computationally much more problematic when there are enumeration definitions in the ontology. In fact, one can expect positive results only for quantifier-free CQs, and even then the required constructions are quite subtle.

Theorem 10. *Given an OMQ $Q = (\mathcal{O}, q)$ with $\mathcal{O}$ a schema.org-ontology and q a quantifier-free CQ, one can construct in polynomial time a datalog-rewriting of Q. Moreover, evaluating OMQs from this class is in* PTIME *in combined complexity. The rewriting is non-recursive if $q = A(x)$.*

The following example illustrates the construction of the datalog program. Let $\mathcal{O} = \{A \equiv \{a_1, a_2\}\}$ and $q() = r(a_1, a_2)$. Observe that $\mathcal{O}, \mathcal{A}'_m \models q()$ for every data instance $\mathcal{A}'_m$ defined in Figure 1b. Similarly to Example 5, one can use the data instances $\mathcal{A}'_m$ to show that $(\mathcal{O}, q())$ is not FO-rewritable.

A datalog-rewriting of $(\mathcal{O}, q())$ is given by the program Π_{a_1,a_2} which contains

$$\begin{aligned}
\mathsf{goal}() &\leftarrow r(a_1, a_2)\\
\mathsf{goal}() &\leftarrow r(a_1, x) \wedge \mathsf{path}_A(x, y) \wedge r(y, a_2)\\
\mathsf{path}_A(x, y) &\leftarrow r(x, y) \wedge A(x) \wedge A(y)\\
\mathsf{path}_A(x, y) &\leftarrow \mathsf{path}_A(x, z) \wedge \mathsf{path}_A(z, y).
\end{aligned}$$

It is also instructive to check that $\mathcal{O}', \mathcal{A}'_m \not\models q()$ with $\mathcal{O}' = \{A \equiv \{a_1, a_2, a_3\}\}$ because in models of $\mathcal{O}'$, a_3 can be identified with some b_i, a_1 with $b_1, \dots, b_{i-1}$ and a_2 with $b_{i+1}, \dots, b_m$, $1 \leq i \leq m$.

We now modify the datalog program above to obtain a rewriting of the OMQ $(\mathcal{O}, q'(x, y))$ with $q(x, y) = r(x, y)$. First, we include in Π_r the rules $A(a_1) \leftarrow \mathsf{true}$ and $A(a_2) \leftarrow \mathsf{true}$. Then we add the following rules:

$$\mathsf{goal}(x, y) \leftarrow r(x, y), \qquad \mathsf{goal}(x, y) \leftarrow A(x) \wedge A(y) \wedge \bigwedge_{1 \leq i,j \leq 2} R_{a_i,a_j}(x, y).$$

We want to use the latter rule to check that x, y have to be mapped to $\{a_1, a_2\}$, and that for every possible assignment a_i, a_j to x, y that is consistent (i.e., we do not have $x \in \{a_1, a_2\}$ and $x \neq a_i$, and similarly for y), $r(a_i, a_j)$ is true. To this end, we add the rules:

$$\begin{aligned}
R_{a_i,a_j}(x, y) &\leftarrow \mathsf{neq}(x, a_i) \qquad & R_{a_i,a_j}(x, y) &\leftarrow \mathsf{neq}(y, a_j)\\
R_{a_i,a_j}(x, y) &\leftarrow \mathsf{goal}(a_i, a_j) & &\\
\mathsf{neq}(a_1, a_2) &\leftarrow \mathsf{true} & \mathsf{neq}(a_2, a_1) &\leftarrow \mathsf{true}.
\end{aligned}$$

It remains to add rules 3 and 4 from Π_{a_1,a_2} and

$$\mathsf{goal}(a_i, a_j) \leftarrow r(a_i, x) \wedge \mathsf{path}_A(x, y) \wedge r(y, a_j)$$

for $1 \leq i, j \leq 2$ and $i \neq j$.

Theorem 10 is tight in the sense that evaluating CQs with a single atom and a single existentially quantified variable, as well as quantifier-free UCQs, is coNP-hard in data complexity. For instance, let $\mathcal{O} = \{\mathsf{dom}(e) \sqsubseteq A,\ \mathsf{ran}(e) \sqsubseteq A,\ A \equiv \{r, g, b\}\}$. Then, an undirected graph $G = (V, E)$ is 3-colorable iff $\mathcal{O}, \{e(v, w) \mid (v, w) \in E\} \not\models \exists x\, e(x, x)$. Alternatively, one may replace the query by $r(r, r) \vee r(g, g) \vee r(b, b)$. In fact, one can prove the following variant of Theorem 5 which shows that classifying OMQs with ontologies using *only* enumeration definitions and quantifier-free UCQs according to their complexity is as hard as CSP.

Theorem 11. *Given a template $\mathcal{B}$, one can construct in polynomial time an OMQ $(\mathcal{O}, q)$ where $\mathcal{O}$ only contains enumeration definitions and q is a Boolean variable-free UCQ such that the complement of CSP($\mathcal{B}$) and $(\mathcal{O}, q)$ are mutually FO-reducible.*

We now turn to classifying the complexity of ontologies and of OMQs, starting with a generalization of Theorem 4 to unrestricted schema.org-ontologies.

Theorem 12. *Let $\mathcal{O}$ be a coherent and minimized schema.org-ontology. If $\mathcal{O}$ contains an enumeration definition $A \equiv \{a_1, \dots, a_n\}$ with $n \geq 2$ or contains an inclusion $F \sqsubseteq A_1 \sqcup \dots \sqcup A_n$ such that there are at least two concept names in $\{A_1, \dots, A_n\}$ and $\mathcal{O} \not\models F \sqsubseteq A \sqcup \bigsqcup_{(D,\Delta^{\mathcal{D}}) \in DT} D$ for any A with $A \equiv \{a\} \in \mathcal{O}$, then $(\mathcal{O}, q)$ is coNP-hard for some Boolean CQ q. Otherwise every $(\mathcal{O}, q)$ with q a UCQ is FO-rewritable (and thus in AC^0 in data complexity).*

Note that, in contrast to Theorem 4, being in AC^0 does not mean that no 'real disjunction' is available. For example, for $\mathcal{O} = \{\mathsf{ran}(r) \sqsubseteq A \sqcup B, A \sqsubseteq C, B \sqsubseteq C, C \equiv \{c\}\}$ and $\mathcal{A} = \{r(a, b)\}$ we have $\mathcal{O}, \mathcal{A} \models A(b) \vee B(b)$ and neither $A(b)$ nor $B(b)$ are entailed. This type of choice does not affect FO-rewritability, however, since it is restricted to individuals that must be identified with a unique individual in $\mathsf{N_E}(\mathcal{O})$. Note that, for the hardness proof, we now need to use a role name that possibly does not occur in $\mathcal{O}$. For example, for $\mathcal{O} = \{A \equiv \{a_1, a_2\}\}$ there exists a Boolean CQ q such that $(\mathcal{O}, q)$ is NP-hard, but constructing q requires a fresh role name.

We now consider the complexity of single OMQs and show a converse of Theorems 5 and 11 for schema.org-ontologies and UCQs that are *qvar-acyclic*, that is, when all atoms $r(t, t')$ with neither of t, t' a quantified variable are dropped, then all CQs in it are acyclic. We use *generalized CSPs with marked elements* in which instead of a single template $\mathcal{B}$, one considers a finite set Γ of templates whose signature contains, in addition to concept and role names, a finite set of individual names. Homomorphisms have to respect also the individual names and the problem is to decide whether there is a homomorphism from the input interpretation to some $\mathcal{B} \in \Gamma$. Every such CSP is mutually FO-reducible with some standard CSP and FO-definability and datalog definability of the complement of generalized CSPs with marked elements are NP-complete Bienvenu *et al.* [2014b].

Theorem 13. *Given an OMQ $(\mathcal{O}, q)$ with $\mathcal{O}$ a schema.org-ontology and q a qvar-acyclic UCQ, one can compute in exponential time a generalized CSP with marked elements Γ such that $(\mathcal{O}, q)$ and the complement of CSP(Γ) are mutually FO-reducible.*

The proof uses an encoding of qvar-acyclic queries into concepts in the description logic $\mathcal{ALCIUO}$ that extends $\mathcal{ALC}$ by inverse roles, the universal role, and nominals. It extends the the template constructions of Bienvenu *et al.* [2014b] to description logics with nominals. As a particularly interesting consequence of Theorem 13, we obtain:

Theorem 14. *FO-rewritability and datalog-rewritability of OMQs $(\mathcal{O}, q)$ with $\mathcal{O}$ a schema.org-ontology and q a qvar-acyclic UCQ are decidable in* NEXPTIME.

6 Conclusion

The work presented in this paper lays a solid foundation for attacking many interesting and practically relevant questions that can be asked about querying in the presence of schema.org-ontologies. Topics of interest include different forms of queries such as SPARQL and regular path queries as well as uncertainty in the data that accounts for varying levels of trust in different data sources.

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